QUESTION PAPER WITH SOLUTION

PHYSICS _ 4 Sep. _ SHIFT - 1

Q.1 Starting from the origin at time t = 0, with initial velocity $5\hat{j}\,ms^{-1}$, a particle moves in the x-y plane with a constant acceleration of $\left(10\hat{i}+4\hat{j}\right)ms^{-2}$. At time t, its coordinates are (20 m, y_0 m). The values of t and y_0 are, respectively:

(1) 5s and 25 m (2) 2s and 18 m (3) 2s and 24 m (4) 4s and 52 m

Sol. **2**

$y = u_y t + \frac{1}{2} a_y t^2$

$y = 5t + \frac{1}{2}(4)\,t^2$

$y = 5t + 2t^2$

and $x = 0 + \frac{1}{2}(10)(t^2) = 20$

$t = 2$ s

$\Rightarrow y = 10 + 8 = 18$m

Q.2 A small bar magnet placed with its axis at 30° with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilbrium position is:

(1) 7.2×10^{-2} J (2) 6.4×10^{-2} J (3) 9.2×10^{-3} J (4) 11.7×10^{-3} J

Sol. **1**

$\tau = MB \sin 30^\circ$

$0.018 = MB\left(\frac{1}{2}\right)$

$MB = 0.036$

$w = \Delta U = 2MB = 0.072$ J

Q.3 Choose the correct option relating wave lengths of different parts of electromagnetic wave spectrum:

(1) $\lambda_{\text{radio waves}} > \lambda_{\text{micro waves}} > \lambda_{\text{visible}} > \lambda_{\text{x-rays}}$ (2) $\lambda_{\text{visible}} > \lambda_{\text{x-rays}} > \lambda_{\text{radio waves}} > \lambda_{\text{micro waves}}$

(3) $\lambda_{\text{visible}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{x-rays}}$ (4) $\lambda_{\text{x-rays}} < \lambda_{\text{micro waves}} < \lambda_{\text{radio waves}} < \lambda_{\text{visible}}$

Sol. **1**

By property of electromagnetic wave spectrum.

Q.4 On the x-axis and at a distance x from the origin, the gravitational field due a mass distribution is given by $\frac{Ax}{\left(x^2+a^2\right)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is:

(1) $A\left(x^2+a^2\right)^{3/2}$ (2) $\frac{A}{\left(x^2+a^2\right)^{1/2}}$ (3) $A\left(x^2+a^2\right)^{1/2}$ (4) $\frac{A}{\left(x^2+a^2\right)^{3/2}}$

Sol. **2**

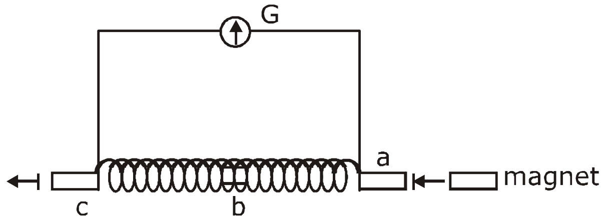
$E_x = \dfrac{Ax}{(x^2+a^2)^{3/2}}$

$\dfrac{-dv}{dx} = \dfrac{Ax}{(x^2+a^2)^{3/2}}$

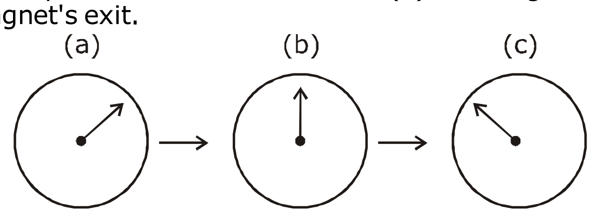
$\int_0^V dv = -\int_\infty^x \dfrac{Ax}{(x^2+a^2)^{3/2}}\,dx$

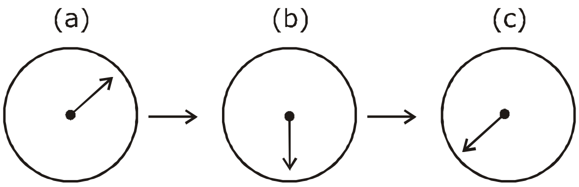
$V = \dfrac{A}{(x^2+a^2)^{1/2}}$

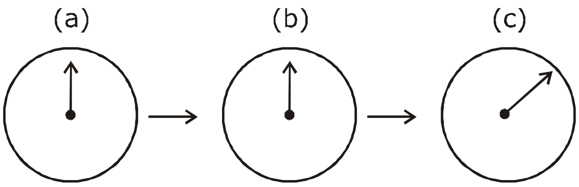
Q.5 A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer G attached across the coil?

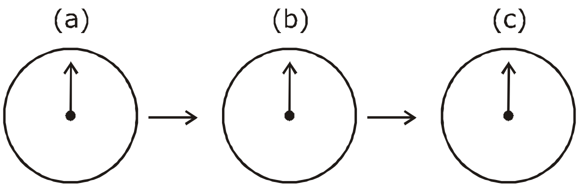


Three positions shown describe: (a) the magnet's entry (b) magnet is completely inside and (c) magnet's exit.

(1)


(2)


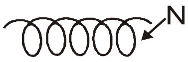
(3)


(4)


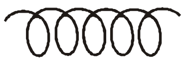
Sol. **1**

Let N S

→ When bar magnet enter

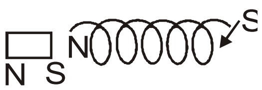


→ When completely inside



i = 0

→ when exit



Q.6 A battery of 3.0V is connected to a resistor dissipating 0.5 W of power. If the terminal voltage of the battery is 2.5V, the power dissipated within the internal resistance is:

(1) 0.072 W (2) 0.10 W (3) 0.125 W (4) 0.50 W

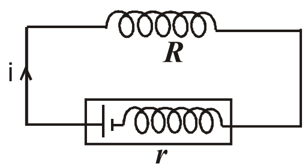
Sol. **2**

$P_0 = 0.5$ w

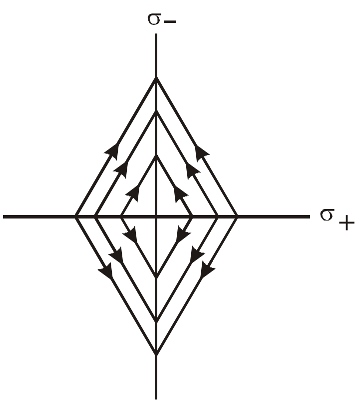
i. (2.5) = 0.5

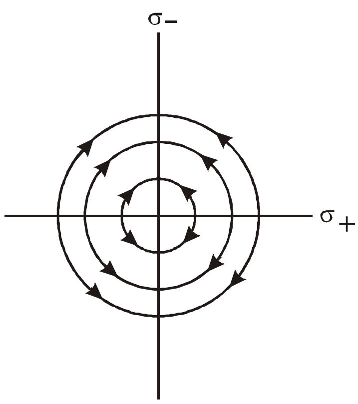
i = 1/5 A

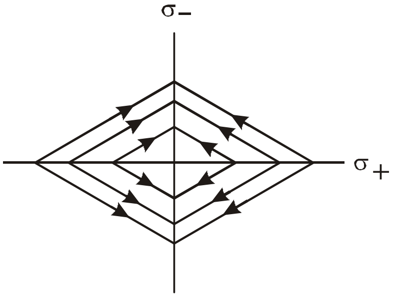
$P_r = \left(\frac{1}{5}\right)(0.5) = 0.1$ W

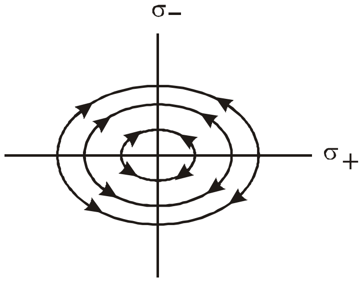


Q.7 Two charged thin infinite plane sheets of uniform surface charge density σ_+ and σ_-, where $|\sigma_+| > |\sigma_-|$, intersect at right angle. Which of the following best represents the electric field lines for this system:

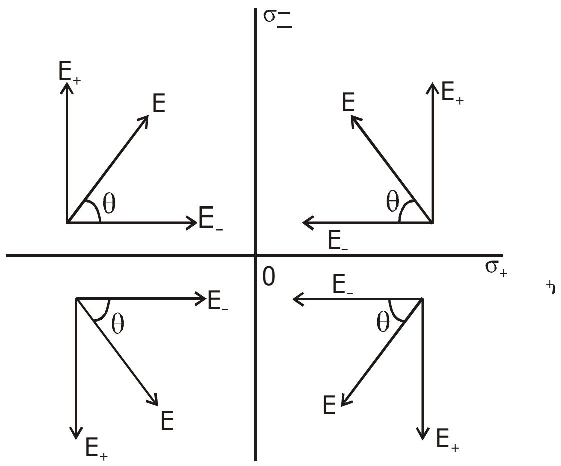
(1)


(2)


(3)


(4)


Sol. **1**



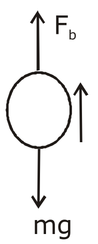
$\left|\vec{E}_{+}\right| > \left|\vec{E}_{-}\right|$

$\theta > 45°$

Q.8 A air bubble of radius 1 cm in water has an upward acceleration 9.8 cm s^{-2}. The density of water is 1 gm cm^{-3} and water offers negligible drag force on the bubble. The mass of the bubble is (g = 980 cm/s^2).

(1) 1.52 gm (2) 4.51 gm (3) 3.15 gm (4) 4.15 gm

Sol. **4**

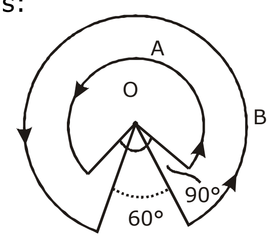


$F_b - mg = ma \qquad \Rightarrow m = \frac{F_b}{g + a}$

$m = \frac{V \cdot \rho_w g}{g + a}$

$m = \frac{(4/3)\pi r^3 . \rho_w . g}{g + a} = 4.15gm$

Q.9 A wire A, bent in the shape of an arc of a circle, carrying a current of 2A and having radius 2 cm and another wire B, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic field due to the wires A and B at the common centre O is:



(1) 2 : 5 (2) 6 : 5 (3) 6 : 4 (4) 4 : 6

Sol. **2**

$$B_A = \frac{\mu(2)\left(\frac{3\pi}{2}\right)}{2(a)(2\pi)} = \frac{3\mu}{4a}$$

$$B_B = \frac{\mu(3)\left(\frac{5\pi}{3}\right)}{2(2a)(2\pi)} = \frac{5\mu}{8a}$$

$$\frac{B_A}{B_B} = \frac{3\mu}{4a} \times \frac{8a}{5\mu} = 6:5$$

Q.10 Particle A of mass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. If both particles move along the x-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavlength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is:

(1) $\Delta\lambda = \frac{5}{2}\lambda_0$ (2) $\Delta\lambda = 2\lambda_0$ (3) $\Delta\lambda = 4\lambda_0$ (4) $\Delta\lambda = \frac{3}{2}\lambda_0$

Sol. **3**

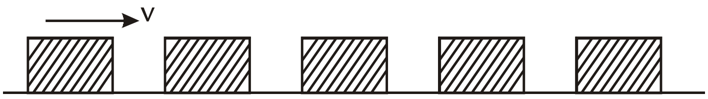
$$V_1 = \frac{m_1 - m_2}{m_1 + m_2} . u_1 + \frac{2m_2}{m_1 + m_2} . u_2$$

$$V_1 = \frac{\frac{m}{2} - m/3}{\frac{m}{2} + m/3} V_0 = V_0/5$$

$$\lambda' = \frac{h}{\frac{m}{2}.\frac{V_0}{5}} = 5.\frac{h}{\frac{m}{2}.V_0} = 5\lambda_0$$

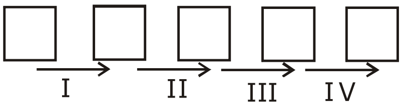
$\Delta\lambda = 4\lambda_0$

Q.11 Blocks of masses m, 2m, 4m and 8m are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass 8m starts moving the total energy loss is p% of the original energy. Value of 'p' is close to:



(1) 94 (2) 87 (3) 37 (4) 77

Sol. **1**

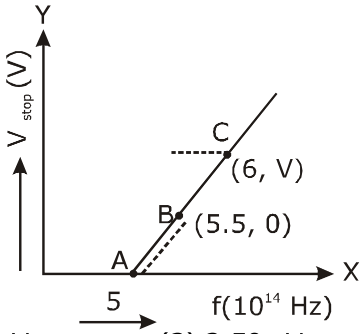


There will be total 4 collisions in each collision K.E. decreasing by 50%

$E_f = \frac{1}{2^4} E_i = \frac{E_i}{16}$ = 6.25%

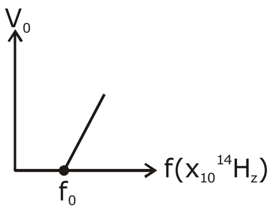
i.e. 93.75 % loss

Q.12 Given figure shows few data points in a phot electric effect experiment for a certain metal. The minimum energy for ejection of electron from its surface is: (Plancks constant h = 6.62 × 10^{-34}J.s)



(1) 2.10 eV (2) 2.27 eV (3) 2.59 eV (4) 1.93 eV

Sol. **2**



$\phi = hf_0 = 6.62 \times 10^{-34} \times 5.5 \times 10^{14}$

$= 36.41 \times 10^{-20}$J = 2.27 eV

Q.13 The specific heat of water = 4200 J $kg^{-1}K^{-1}$ and the latent heat of ice = 3.4×10^5 J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams):

(1) 63.8 (2) 64.6 (3) 61.7 (4) 69.3

Sol. **3**

Heat loss by water

$Q = m_w s\Delta\theta$

$= \left(\frac{200}{1000}\right)$. (4200) (25) = 21000 J

and $\Delta m_i L = 21000$

$\Delta m_i = \frac{21000}{3.4\times10^5}\times10^3\, gm$ = 61.7 grams

Q.14 A beam of plane polarised light of large cross-sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional area 3×10^{-4} m^2) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to:

(1) 1.0×10^{-4} J (2) 1.0×10^{-5} J (3) 5.0×10^{-4} J (4) 1.5×10^{-4} J

Sol. **1**

$p = p_0 \cos^2\omega t$

$E_{avg} = \langle p \rangle \cdot T = \frac{p_0}{2}T$

$E_{avg} = \langle P \rangle . T = \frac{p_0}{2}.\frac{2\pi}{\omega} = \frac{10^{-3}\times3.14}{31.4} = 10^{-4}J$

Q.15 For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5m, while the distance between one crest and one trough is 1.5m. The possible wavelengths (in m) of the waves are:

(1) 1, 3, 5,..... (2) 1, 2, 3,..... (3) $\frac{1}{2},\frac{1}{4},\frac{1}{6},....$ (4) $\frac{1}{1},\frac{1}{3},\frac{1}{5},....$

Sol. **4**

$1.5 = (2n_1 + 1)\lambda/2$ ...(1)

$5 = n_2\lambda$ ...(2)

n_1 & n_2 are integer

$n_1 = 1$, $n_2 = 5$

$n_1 = 2$, n_2 is not integer

$n_1 = 3$, n_2 is not integer

$n_1 = 4$, $n_2 = 15$, $\lambda = 1/3$

Q.16 Match the C_p/C_V ratio for ideal gases with different type of molecules:

Molecule Type	C_p/C_V
(A) Monoatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(2) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(3) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Sol. **4**

$\gamma = C_p/C_v$

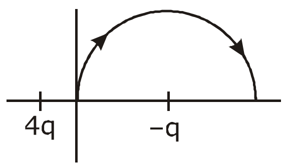
$\gamma_A = 1 + \frac{2}{3} = 5/3$

$\gamma_B = 1 + \frac{2}{5} = 7/5$

$\gamma_C = 1 + \frac{2}{7} = 9/7$

$\gamma_d = 1 + \frac{2}{6} = 4/3$

Q.17 A two point charges 4q and –q are fixed on the x-axis at $x = -\frac{d}{2}$ and $x = \frac{d}{2}$, respectively. If a third point charge 'q' is taken from the origin to x = d along the semicircle as shown in the figure, the energy of the charge will:



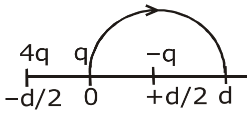
(1) decrease by $\frac{q^2}{4\pi \epsilon_0 d}$

(2) decrease by $\frac{4q^2}{3\pi \epsilon_0 d}$

(3) increase by $\frac{3q^2}{4\pi \epsilon_0 d}$

(4) increase by $\frac{2q^2}{3\pi \epsilon_0 d}$

Sol. **2**



$\Delta U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q.q}{(3d/2)} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{4q.q}{(d/2)}$

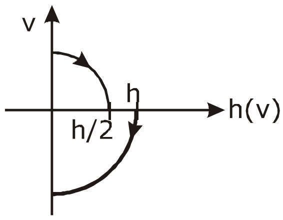
$= \frac{4q^2}{4\pi\varepsilon_0}\left(\frac{2}{d}\right)\left(-\frac{2}{3}\right)$

$= (-)\ \frac{4q^2}{3\pi\varepsilon_0 .d}$

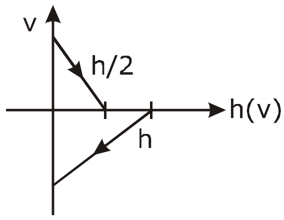
= decrease by (−)

Q.18 A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by: (graph are drawn schematically and on not to scale)

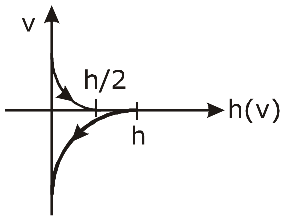
(1)



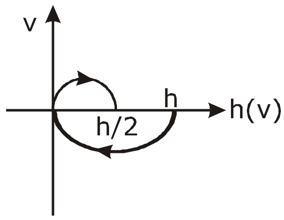
(2)



(3)



(4)



Sol. **1**

$\rightarrow$ V, h curve will be parabolic

$\rightarrow$ downward velocity is negative and upward is positive

$\rightarrow$ when ball is coming down graph will be in IV quadrant and when going up graph will be in I quadrant

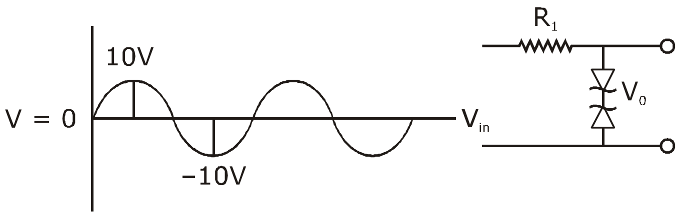
Q.19 Dimensional formula for thermal conductivity is (here K denotes the temperature):

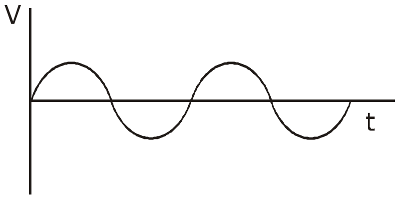
(1) $MLT^{-3}K^{-1}$ (2) $MLT^{-2}K^{-2}$ (3) $MLT^{-2}K$ (4) $MLT^{-3}K$

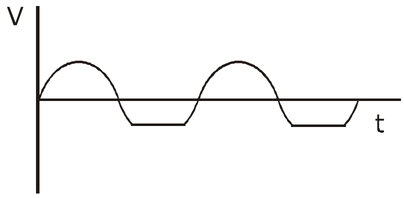
Sol. **1**

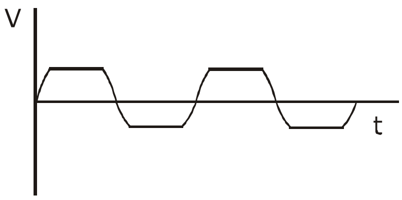
$\frac{dQ}{dt} = \frac{Kl\Delta T}{A}$

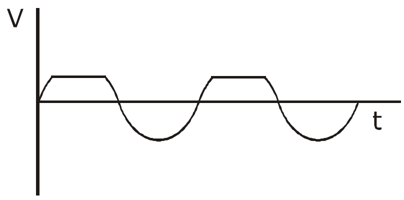
Q.20 Take the breakdown voltage of the zener diode used in the given circuit as 6V. For the input voltage shown in figure below, the time variation of the output voltage is : (Graphs drawn are schematic and not to scale)



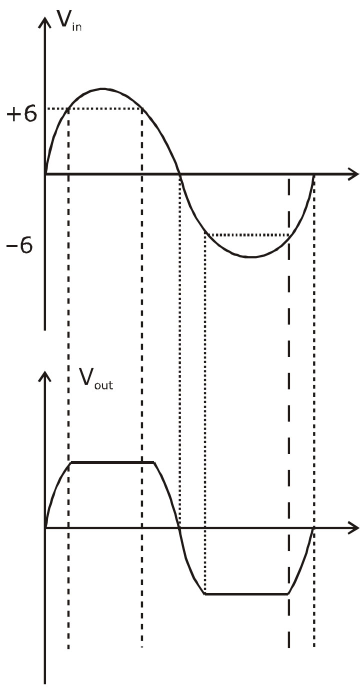
(1)


(2)


(3)


(4)


Sol. **3**



Q.21 In the line spectra of hydrogen atoms, difference between the largest and the shortest wavelengths of the Lyman series is 304Å. The corresponding difference for the Paschan series in Å is : ___________.

Sol. **10553**

$\frac{1}{R}$ = 912 Å

in Paschen series

$$\frac{1}{\lambda_s} = R\left(\frac{1}{3^2}\right) = \frac{R}{9}$$

$$\frac{1}{\lambda_l} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$$

$$(\lambda_l - \lambda_s) = \left(\frac{144}{7} - 9\right)R = 10553 \text{ Å}$$

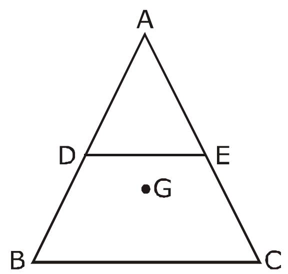
Q.22 A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to _______.

Sol. **266**

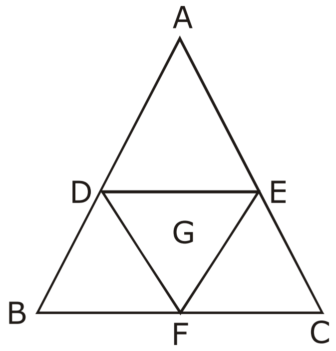
$$(0.1)\left(\frac{3}{2}R\right)(T-200) = (0.05)\left(\frac{3}{2}R\right)(400-T)$$

T = 266.6 K

Q.23 ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is ______________.



Sol. **11**



Let $I_0 = kmc^2$

$I_{DEF} = K\left(\frac{m}{\ell}\right)\left(\frac{\ell}{2}\right)^2 = \left(\frac{I_0}{16}\right)$

and $I_{ADE} = I_{BDE} = I_{EFC} = I$

$3I = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$

$\Rightarrow I = \frac{5I_0}{16}$

$I_{remaining} = 2I + \frac{I_0}{16} = \frac{11I_0}{16}$

Q.24 In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is __________.

Sol. **6.25**

$L = 20$, $f_0 = 1cm$, $M = 100$

$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$ $\quad [v_0 \approx L,\ u_0 \approx f_0]$

on solving we get

$f_e = 6.25$ cm

Q.25 A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is droped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2 the energy lost in the process is p% of the initial energy. Value of p is ___________.

Sol. **20**

$I_f \omega_f = I_i \omega_i$

$I_i = \frac{MR^2}{2}$

$I_f = \frac{MR^2}{2} + \frac{M(R/2)^2}{2}$

$= \frac{5}{4} \cdot \frac{MR^2}{2}$

$\left[\frac{MR^2}{2} + \frac{M}{2}\left(\frac{R}{2}\right)^2\right]\omega' = \left(\frac{MR^2}{2}\right).\omega$

$\left[\frac{MR^2}{2} \cdot \left(\frac{5}{4}\right)\right]\omega' = \frac{MR^2}{2}\omega$

$\omega' = \frac{4}{5}\omega$

loss of K.E. $= \frac{Loss}{K_i} \times 100 = \frac{\omega^2 - \omega'^2 (5/4)}{\omega^2} \times 100$

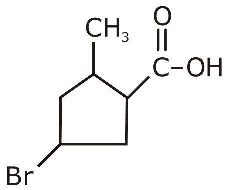
$\frac{\omega^2 - \frac{16}{25}\omega^2\left(\frac{5}{4}\right)}{\omega^2} = \left(1 - \frac{80}{100}\right) \times 100$

$= 20\%$

QUESTION PAPER WITH SOLUTION

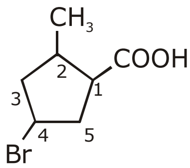
CHEMISTRY _ 4 Sep. _ SHIFT - 1

1. The IUPAC name of the following compound is :



(1) 3-Bromo-5-methylcyclopentane carboxylic acid
(2) 4-Bromo-2-methylcyclopentane carboxylic acid
(3) 5-Bromo-3-methylcyclopentanoic acid
(4) 3-Bromo-5-methylcyclopentanoic acid

Sol. **2**



4–Bromo-2-methylcyclopentane carboxylic acid

2. On heating, lead(II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with NO changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is :

(1) +3 (2) +4 (3) +2 (4) +5

Sol. **1**

$Pb(NO_3)_2 \rightarrow PbO + \underset{\text{Brown gas}}{\underset{(A)}{NO_2}} + O_2$

$2NO_2 \xrightarrow{\text{cooling}} \underset{\text{colourless solid}}{\underset{(C)}{N_2O_4}}$

$NO_2 + NO \rightarrow \underset{\text{blue solid}}{\underset{(C)}{N_2O_3}} + \overset{+3}{N_2}O_3$

3. The ionic radii of O^{2-}, F^-, Na^+ and Mg^{2+} are in the order :

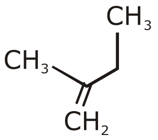
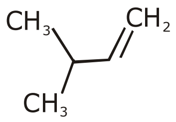
(1) $F^- > O^{2-} > Na^+ > Mg^{2+}$
(2) $Mg^{2+} > Na^+ > F^- > O^{2-}$
(3) $O^{2-} > F^- > Na^+ > Mg^{2+}$
(4) $O^{2-} > F^- > Mg^{2+} > Na^+$

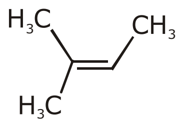
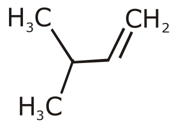
Sol. **3**

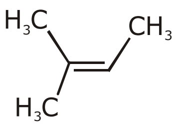
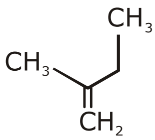
$O^{2-} > F^- > Na^{\oplus} > Mg^{2+}$

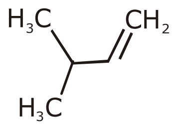
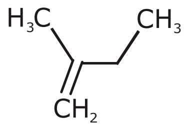
Ans. option (3)

4. When neopentyl alcohol is heated with an acid, it slowly converted into an 85 : 15 mixture of alkenes A and B, respectively. What are these alkenes ?

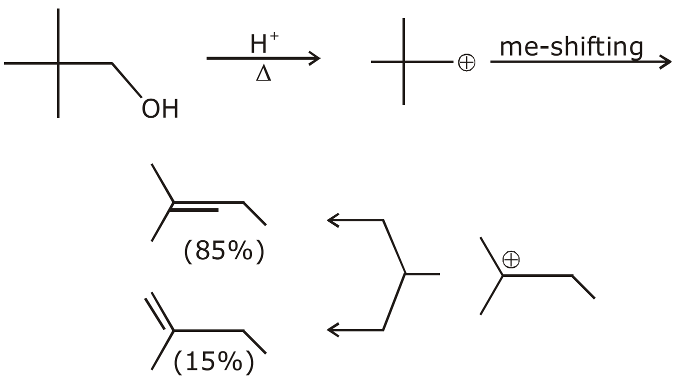
(1)  and 

(2)  and 

(3)  and 

(4)  and 

Sol. **3**



5. The region in the electromagnetic spectrum where the Balmer series lines appear is :

(1) Microwave (2) Infrared (3) Ultraviolet (4) Visible

Sol. **4**

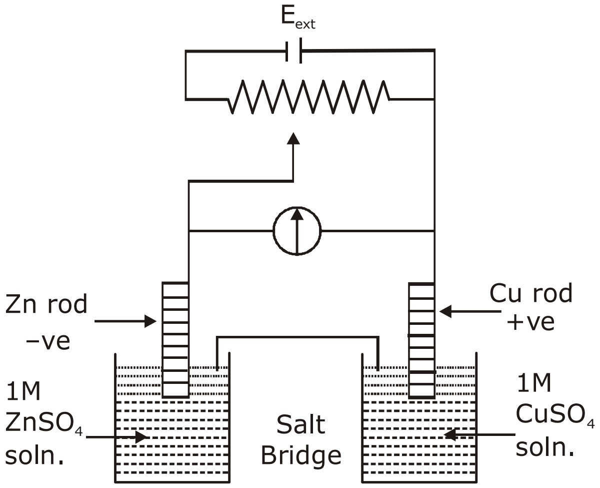
Question should be Bonous

As lines of Balamer series belongs to both UV as well visible region of EM spectrum.

However most appropriate should be visible region

Ans. (4)

6.



$E^{o}_{Cu^{2+}|Cu} = +\ 0.34\ V$

$E^{o}_{Zn^{2+}|Zn} = -\ 0.76\ V$

Identify the incorrect statement form the options below for the above cell :

(1) If E_{ext} = 1.1 V, no flow of e^- or current occurs

(2) If E_{ext} > 1.1 V, Zn dissolves at Zn electrode and Cu deposits at Cu electrode

(3) If E_{ext} > 1.1 V, e^- flows from Cu to Zn

(4) If E_{ext} < 1.1 V, Zn dissolves at anode and Cu doposits at cathode

Sol. **2**

Direction NCERT Text theoritical questions

Ans. (2)

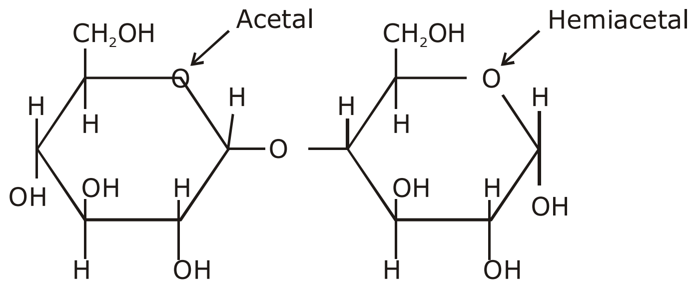
7. What are the functional groups present in the structure of maltose ?

(1) One acetal and one hemiacetal (2) One acetal and one ketal

(3) One ketal and one hemiketal (4) Two acetals

Sol. **1**

Meltose



8. Match the following :

(i) Foam	(a) smoke
(ii) Gel	(b) cell fluid
(iii) Aerosol	(c) jellies
(iv) Emulsion	(d) rubber
	(e) froth
	(f) milk

(1) (i)-(e), (ii)-(c), (iii)-(a), (iv)-(f)
(2) (i)-(b), (ii)-(c), (iii)-(e), (iv)-(d)
(3) (i)-(d), (ii)-(b), (iii)-(a), (iv)-(e)
(4) (i)-(d), (ii)-(b), (iii)-(e), (iv)-(f)

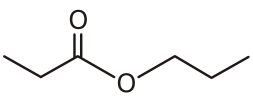
Sol. **1**

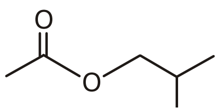
Foam	$\rightarrow$	Froth, whipped cream, soaplather
Gel	$\rightarrow$	Cheese, butter, jellies
Aerosol	$\rightarrow$	smoke dust
Emulsion	$\rightarrow$	milk
Sol	$\rightarrow$	Cell fluid
rubber	$\rightarrow$	Solid fom
froth	$\rightarrow$	form

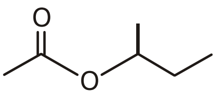
(i) - e, (ii) - c, (iii) - a, (iv) - f

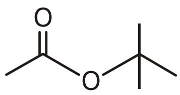
Ans. 1

9. An organic compound (A) (molecular formula $C_6H_{12}O_2$) was hydrolysed with dil. H_2SO_4 to give a carboxylic acid (B) and an alcohol (C). 'C' gives white turbidity immediately when treated with anhydrous $ZnCl_2$ and conc. HCl. The organic compound (A) is :

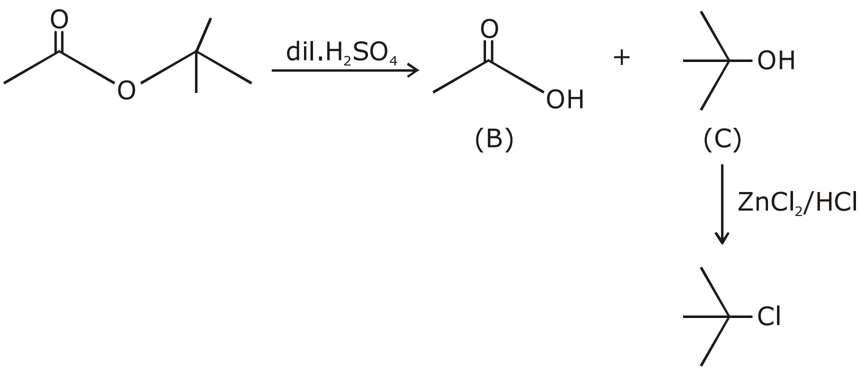
(1) 

(2) 

(3) 

(4) 

Sol. **4**



(Immidiate white turbidity)

10. Among the statements (a)-(d), the correct ones are :

(a) Lime stone is decomposed to CaO during the extraction of iron from its oxides.

(b) In the extraction of silver, silver is extracted as an anionic complex.

(c) Nickel is purified by Mond's process.

(d) Zr and Ti are purified by Van Arkel method.

(1) (c) and (d) only
(2) (b), (c) and (d) only
(3) (a), (b), (c) and (d)
(4) (a), (c) and (d) only

Sol. **3**

Lime stone finally goes to slag formation

$CaCO_3 \rightarrow CaO + CO_2$

$CaO + SiO_2 \rightarrow \underset{\text{slag}}{CaSiO_3}$

11. For one mole of an ideal gas, which of these statements must be true ?

(a) U and H each depends only on temperature

(b) Compressibility factor z is not equal to 1

(c) $C_{P,\,m}-C_{V,\,m}=R$

(d) $dU=C_V dT$ for any process

(1) (a), (c) and (d)
(2) (a) and (c)
(3) (c) and (d)
(4) (b), (c) and (d)

Sol. **1**

For ideal gas

$$\left.\frac{\delta v}{\delta v}\right|_T = 0 \ \& \ \left.\frac{\delta H}{\delta v}\right|_T = 0$$

(a) Hence function of temp. only.

(b) Compressibility factor (z) = 1 Always

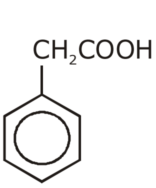
(c) $C_{p,m} - C_{V,m} = R$

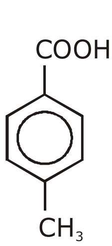
(d) $dv = nC_{v,m}\, dT$ for all process

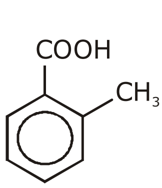
Ans. a,c,d

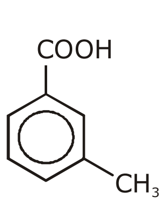
option (1)

12. [P] on treatment with $Br_2/FeBr_3$ in CCl_4 produced a single isomer $C_8H_7O_2Br$ while heating [P] with sodalime gave toluene. The compound [P] is :

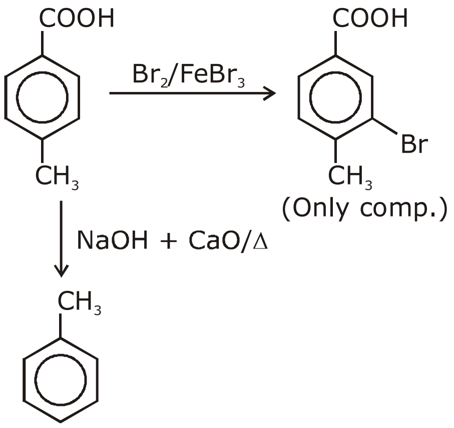
(1) 

(2) 

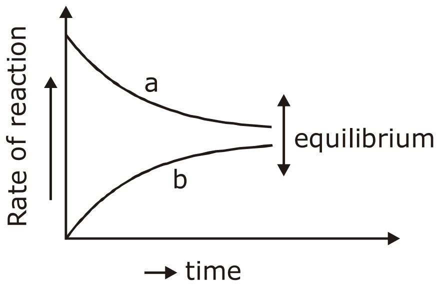
(3) 

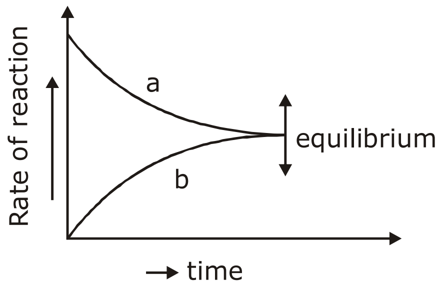
(4) 

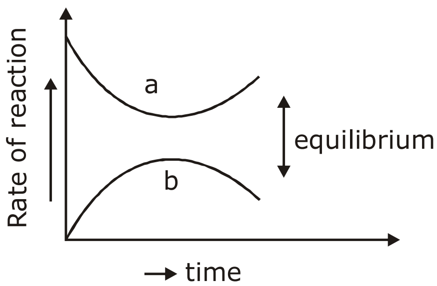
Sol. 2

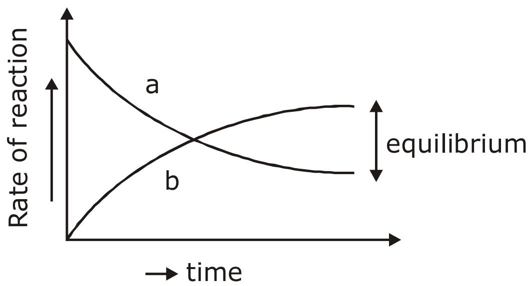


13. For the equilibrium A $\rightleftharpoons$ B the variation of the rate of the forward (a) and reverse (b) reaction with time is given by :

(1)


(2)


(3)


(4)


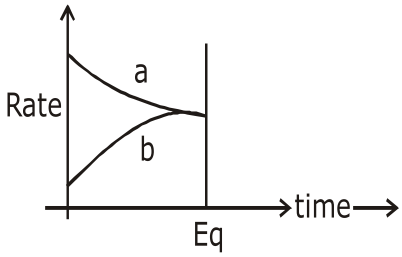
Sol. **2**

At equilibrium

Rate of forward = Rate of backward

a = b

Hence



Ans. option (2)

14. The pair in which both the species have the same magnetic moment (spin only) is :

(1) $[Co(OH)_4]^{2-}$ and $[Fe(NH_3)_6]^{2+}$
(2) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)]^{2+}$
(3) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$
(4) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

Sol. **4**

$[Cr(H_2O)_6]^{2+}$ $3d^4$

$[Fe(H_2O)_6]^{2+}$ $3d^6$ ↿ ↿

↿⇂↿ ↿

Both has 4 unpaired electron

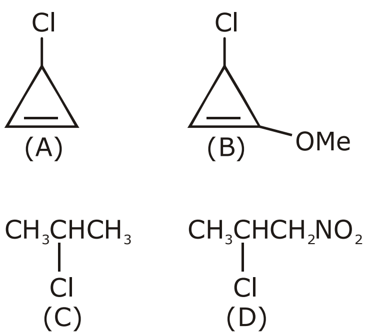
15. The number of isomers possible for $[Pt(en)(NO_2)_2]$ is :

(1) 2 (2) 3 (3) 4 (4) 1

Sol. **2**

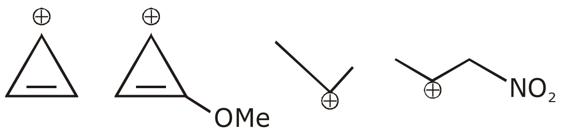
Three linkage isomer NO_2^-; ONO^-

16. The decreasing order of reactivity of the following organic moleules towards $AgNo_3$ solution is :



(1) (B) > (A) > (C) > (D)
(2) (A) > (B) > (C) > (D)
(3) (A) > (B) > (D) > (C)
(4) (C) > (D) > (A) > (B)

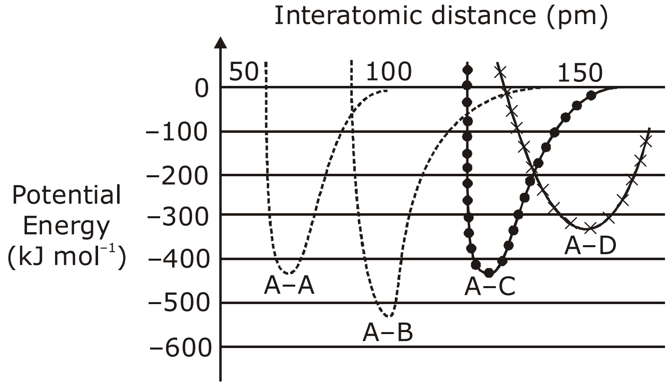
Sol. **1**



(A) (B) (c) (d)

Order or stability
(B) > (A) > (C) > (D)

17. The intermolecular potential energy for the molecules A, B, C and D given below suggests that :



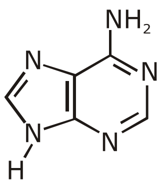
(1) A–A has the largest bond enthalpy.
(2) D is more electronegative than other atoms.
(3) A–D has the shortest bond length.
(4) A–B has the stiffest bond.

Sol. **4**
Acc. to Diagram
Ans option (4)
As E_{A-B} is Highest

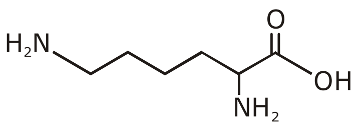
18. Which of the follwong will react with $CHCl_3$ + alc. KOH ?
(1) Thymine and proline
(2) Adenine and thymine
(3) Adenine and lysine
(4) Adenine and proline

Sol. **3**
$CHCl_3$ + Alc. KOH reacts with those compound which have $-NH_2$ group

<u>Adenive</u>



<u>Lysin</u>



19. The elements with atomic numbers 101 and 104 belong to, respectively, :

(1) Actinoids and Group 6
(2) Group 11 and Group 4
(3) Group 6 and Actinoids
(4) Actinoids and Group 4

Sol. **4**

$Z = 101 \rightarrow [R_n]^{86}\, 7s^2\, 5f^{13}$

↓

Actinoids

$Z = 104 \rightarrow [R_n]^{86}\, 7s^2\, 5f^{14}\, 6d^2$

↓

4^{th} group element

Ans Actinoids & 4^{th} group

Ans. (4)

20. On combustion of Li, Na and K in excess of air, the major oxides formed, respectively, are :

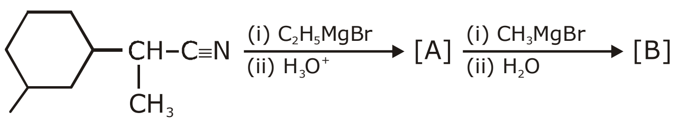
(1) Li_2O_2, Na_2O_2 and K_2O_2
(2) Li_2O, Na_2O_2 and KO_2
(3) Li_2O, Na_2O and K_2O_2
(4) Li_2O, Na_2O_2 and K_2O

Sol. **2**

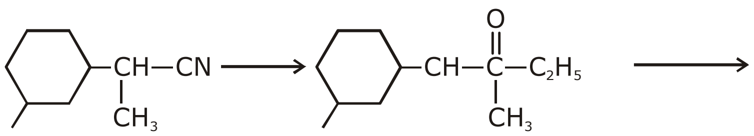
Li_2O, Na_2O_2 K_2O_2

option (2)

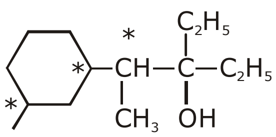
21. The number of chiral centres present in [B] is ________.



Sol. **3**



(A)



3 chiral center is present in final products

22. At 300 K, the vapour pressure of a solution containing 1 mole of n-hexane and 3 moles of n-heptane is 550 mm of Hg. At the same temperature, if one more mole of n-heptane is added to this solution, the vapour pressure of the solution increases by 10 mm of Hg. What is the vapour pressure in mm Hg of n-heptane in its pure state ________?

Sol. $550 = \frac{1}{4} \times p^0_{C_6H_{14}} + \frac{3}{4} \times p^0_{C_7H_{16}}$

$560 = \frac{1}{5} \times p^0_{C_6H_{14}} + \frac{4}{5} \times p^0_{C_7H_{16}}$

$p^0_{C_7H_{16}}$ = [560 × 5 –550 × 4]

= 550 +50 = 600 mm of Hg

23. The mass of ammonia in grams produced when 2.8 kg of dinitrogen quantitatively reacts with 1 kg of dihydrogen is __________.

Sol. $N_2 + 3H_2 \longrightarrow 2NH_3$

2800g 1000g

100 mol 500 mol

L.R.

mole of NH_3 produced = 200 mol

mass = 3400 g

24. If 75% of a first order reaction was completed in 90 minutes, 60% of the same reaction would be completed in approximately (in minutes) _________.

(take : log 2 = 0.30; log 2.5 = 0.40)

Sol. **60**

$t_{75\%}$ = 90 min = 2 × $t_{1/2}$

$t_{1/2}$ = 45 min

$\frac{\ln(2)}{45} \times t_{60\%} = \ln\left\{\frac{100}{40}\right\}$

$t_{60\%} = 45 \times \frac{0.4}{0.3}$

$t_{60\%}$ = 60 min

25. A 20.0 mL solution containing 0.2 g impure H_2O_2 reacts completely with 0.316 g of $KMnO_4$ in acid solution. The purity of H_2O_2 (in %) is ________ (mol. wt. of H_2O_2 = 34′ mole wt. of $KMnO_4$ = 158)

Sol. $H_2O_2 + KmnO_4 \rightarrow Mn^{+2} + O_2$

[moles of H_2O_2] × 2 = $\frac{0.316}{158} \times 5$

moles of H_2O_2 = 5 × 10^{-3}

mass of H_2O_2 = 170 × 10^{-3} g

% purity = $\frac{170 \times 10^{-3}}{0.2} \times 100$ =85%

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 4 Sep. _ SHIFT - 1

1. Let y=y(x) be the solution of the differential equation, $xy'-y=x^2(x\cos x+\sin x), x > 0$. if $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to

(1) $2+\frac{\pi}{2}+\frac{\pi^2}{4}$ (2) $2+\frac{\pi}{2}$ (3) $1+\frac{\pi}{2}$ (4) $1+\frac{\pi}{2}+\frac{\pi^2}{4}$

Sol. **(2)**

$xy' - y = x^2(x \cos x + \sin x)$ $x > 0$, $y(\pi) = \pi$

$y' - \frac{1}{x}y = x\{x\cos x + \sin x\}$

I.F. $= e^{-\int \frac{1}{x}dx} = e^{-\ell nx} = \frac{1}{x}$

$\therefore\ y.\frac{1}{x} = \int \frac{1}{x}.x\left(x\cos x+\sin x\right)dx$

$\frac{y}{x} = \int\left(x\cos x+\sin x\right)dx$

$\frac{y}{x} = \int \frac{d}{dx}\left(x\sin x\right)dx$

$\frac{y}{x} = x\sin x + C$

$\Rightarrow y = x^2 = \sin x + cx$

$x = \pi, y = \pi$

$\pi = \pi c \Rightarrow C = 1$

$y = x^2\sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}+\frac{\pi}{2}$

$y' = 2x \sin x + x^2\cos x + 1$

$y'' = 2\sin x + 2x\cos x + 2x \cos x - x^2\sin x$

$y''\left(\frac{\pi}{2}\right) = 2-\frac{\pi^2}{4} \Rightarrow y\left(\frac{\pi}{2}\right)+y''\left(\frac{\pi}{2}\right) = 2+\frac{\pi}{2}$

2. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:

(1) ${}^{51}C_7 - {}^{30}C_7$ (2) ${}^{51}C_7 + {}^{30}C_7$ (3) ${}^{50}C_7 - {}^{30}C_7$ (4) ${}^{50}C_6 - {}^{30}C_6$

Sol. **(1)**

$\sum_{r=0}^{20} {}^{50-r}C_6$

$\Rightarrow {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + ... + {}^{31}C_6 + {}^{30}C_6$

add and subtract ${}^{30}C_7$

Using
${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r \Rightarrow {}^{30}C_6 + {}^{30}C_7 = {}^{31}C_7$
${}^{31}C_6 + {}^{31}C_7 = {}^{32}C_7$
Similarly solving
${}^{51}C_7 - {}^{30}C_7$

3. Let [t] denote the greatest integer $\leq$ t. Then the equation in x,$[x]^2+2[x+2]-7=0$ has :
(1) exactly four integral solutions.
(2) infinitely many solutions.
(3) no integral solution.
(4) exactly two solutions.

Sol. (2)
$[x]^2 + 2[x + 2] - 7 = 0$
$[x]^2 + 2[x] - 3 = 0$
let $[x] = y$
$y^2 + 3y - y - 3 = 0$
$(y - 1)(y + 3) = 0$
$[x] = 1$ or $[x] = -3$
$x \in [1,2)$ & $x \in [-3,-2)$

4. Let P(3,3) be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at P intersects the x-axis at (9,0) and e is its eccentricity, then the ordered pair (a^2,e^2) is equal to :
(1) (9,3)
(2) $\left(\frac{9}{2},2\right)$
(3) $\left(\frac{9}{2},3\right)$
(4) $\left(\frac{3}{2},2\right)$

Sol. (3)
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ P(3,3)

$\frac{9}{a^2} - \frac{9}{b^2} = 1$ ....(1)

Equation of normal $\Rightarrow \frac{a^2x}{3} + \frac{b^2y}{3} = a^2e^2$

at x – axis $\Rightarrow y = 0$

$\frac{a^2x}{3} = a^2e^2 \Rightarrow x = 3e^2 = 9$
$e^2 = 3$
$e = \sqrt{3}$

$e^2 = 1 + \frac{b^2}{a^2} = 3$
$b^2 = 2a^2$ ...(2)
put in equation 1

$\frac{9}{a^2} - \frac{9}{2a^2} = 1 \Rightarrow \frac{9}{2a^2} = 1 \Rightarrow a^2 = \frac{9}{2}$

$\therefore (a^2, e^2) = \left(\frac{9}{2}, 3\right)$

5. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a>b) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then a^2+b^2 is equal to

(1) 135 (2) 116 (3) 126 (4) 145

Sol. **(3)**

L.R = $\frac{2b^2}{a} = 10$ ...(1)

$\phi(t) = \frac{5}{12} - \left(t - \frac{1}{2}\right)^2 + \frac{1}{4} = \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$

$\therefore \phi(t)_{max} = \frac{2}{3} = e$

$e^2 = 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$

$\frac{b^2}{a.a} = \frac{5}{9}$ from (1)

$\frac{5}{a} = \frac{5}{9} \Rightarrow a = 9$

$\therefore b^2 = 45$

$a^2 + b^2 = 45 + 81 = 126$

6. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx \, (x \ge 0)$. Then f(3) – f(1) is eqaul to :

(1) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$ (3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

Sol. **(4)**

$f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$

$x = \tan^2 t$

$dx = 2\tan t \sec^2 t \, dt$

$$f(x) = \int \frac{\tan t.\ 2\tan t \sec^2 t dt}{\sec^4 t}$$

$$= 2\int \sin^2 t\, dt$$

$x = 3 \Rightarrow t = \frac{\pi}{3}$

$x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\therefore f(3) - f(1) = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1 - \cos 2t)\, dt \Rightarrow \left(t - \frac{1}{2}\sin 2t\right)_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

7. If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+......+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:

(1) (10,97) (2) (11,103) (3) (11,97) (4) (10,103)

Sol. **(2)** 1 + Sn

$T_n = 1 - (2n)^2(2n - 1)$

$= 1 - 4n^2(2n - 1)$

$= 1 - 8n^3 + 4n^2$

$$S_n = \sum_{n=1}^{10} T_n = n - \sum 8n^3 + \sum 4n^2$$

$$= n - 8 \times \frac{n^2(n+1)^2}{4} + \frac{4n(n+1)(2n+1)}{6}$$

$$= 10 - 2 \times 100 \times 121 + \frac{2}{3} \times 10 \times 11 \times 21$$

$= 10 - 24200 + 1540$

$= 10 - 22660$

$\therefore$ Sum of series $= 11 - 22660 = \alpha - 220\beta$

$\alpha = 11, \beta = 103$

8. The integral $\int \left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to

(where C is a constant of integration):

(1) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

(2) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(3) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

(4) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

Sol. **(1)**

$$\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$$

$$\int \underbrace{x\sec x}_{I}.\underbrace{\frac{x\cos x}{(x\sin x+\cos x)^2}}_{II}dx$$

$$x\sec x\left(\frac{-1}{x\sin x+\cos x}\right)+\int\frac{\sec x+x\sec x\tan x}{(x\sin x+\cos x)}dx$$

$$\Rightarrow \frac{-x\sec x}{x\sin x+\cos x}+\int\frac{(\cos x+x\sin x)}{\cos^2 x(x\sin x+\cos x)}dx \Rightarrow \frac{-x\sec x}{x\sin x+\cos x}+\tan x+C$$

9. Let f(x)= |x-2| and g(x) = f(f(x)) ,$x\in[0,4]$. Then $\int_0^3 (g(x)-f(x))dx$ is equal to:

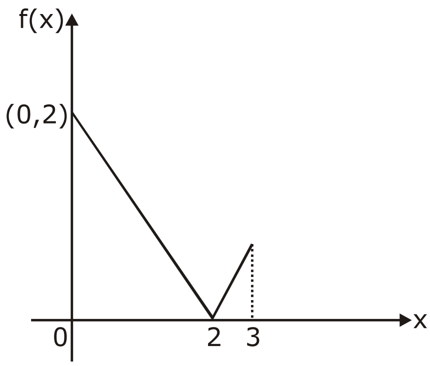
(1) $\frac{1}{2}$ (2) 0 (3) 1 (4) $\frac{3}{2}$

Sol. **(3)**

f(x) = |x – 2|

$$g(x) = ||x-2|-2| = \begin{cases} \text{if } x\ge 2 & \Rightarrow |x-4| \\ \text{if } x<2 & \Rightarrow |-x| \end{cases}$$

$$\therefore \int_0^3 (g(x)-f(x))dx$$



$$= \int_0^3 g(x) - \int_0^3 f(x)\,dx$$

$$= \int_0^2 x\,dx + \int_2^3 (4-x)\,dx - \int_0^2 (2-x)\,dx - \int_2^3 (x-2)\,dx$$

$$\Rightarrow \left(\frac{x^2}{2}\right)_0^2 + \left(4x - \frac{x^2}{2}\right)_2^3 + \left(\frac{x^2}{2} - 2x\right)_0^2 - \left(\frac{x^2}{2} - 2x\right)_2^3$$

$$\Rightarrow 2 + \left\{12 - \frac{9}{2} - 8 + 2\right\} + \{2 - 4\} - \left(\frac{9}{2} - 6 - 2 + 4\right)$$

$$= 2 + \left\{6 - \frac{9}{2}\right\} - 2 - \left\{\frac{9}{2} - 4\right\} = 2 + \frac{3}{2} - \left(2 + \frac{1}{2}\right) = \frac{7}{2} - \frac{5}{2} = 1$$

10. Let x_0 be the point of Local maxima of $f(x) = \vec{a}.(\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ at $x=x_0$ is :

(1) -22 (2) -4 (3) -30 (4) 14

Sol. **(1)**

$$\vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$

$\Rightarrow x\{x^2 - 2\} + 2\{-2x + 7\} + 3\{4 - 7x\}$

$= x^3 - 2x - 4x + 14 + 12 - 21x$

$f(x) = x^3 - 27x + 26$

$f'(x) = 3x^2 - 27 = 0 \Rightarrow x = \pm 3$

Max at $x_0 = -3$

$\therefore \vec{a} = (-3, -2, 3)$, $\vec{b} = (-2, -3, -1)$, $\vec{c} = (7, -2, -3)$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 6 + 6 - 3 - 14 + 6 + 3 - 21 + 4 - 9$

$= 25 - 47 = -22$

11. A triangle ABC lying in the first quadrant has two vertices as A(1,2) and B(3,1) If $\angle BAC = 90^0$, and ar(ΔABC) = $5\sqrt{5}$ s units, then the abscissa of the vertex C is :

(1) $1 + \sqrt{5}$ (2) $1 + 2\sqrt{5}$ (3) $2\sqrt{5} - 1$ (4) $2 + \sqrt{5}$

Sol. **(2)**

$AB = \sqrt{4+1} = \sqrt{5}$

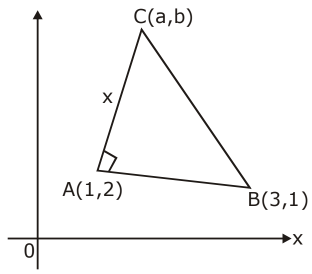
$\frac{1}{2} \times \sqrt{5} \times x = 5\sqrt{5}$

$x = 10$

$m_{AB} = \frac{1}{-2}$

$m_{AC} = 2 = \tan\theta$

$\therefore \sin\theta = \frac{2}{\sqrt{5}}, \cos\theta = \frac{1}{\sqrt{5}}$



by parametric co - ordinates $a = 1 + 10 \times \frac{1}{\sqrt{5}} = 1 + 2\sqrt{5}$

12. Let f be a twice differentiable function on (1,6). If f(2)=8, f′(2)=5, f′(x)≥1 and f‟(x)≥4, for all x∈(1,6), then:

(1) f(5)+f′(5)≥28 (2) f′(5)+f‟(5)≤20

(3) f(5)≤10 (4) f(5)+f′(5)≤26

Sol. **(1)**

$f(2) = 8, f'(2) = 5, f'(x) \geq 1, f''(x) \geq 4$

$x \in (1,6)$

$\int_2^5 f'(x) \geq \int_2^5 1\,dx$

$f(5) - f(2) \geq 3$

$f(5) \geq 11$ ...(1)

also $\int_2^5 f''(x)\,dx \geq \int_2^5 4dx$

$f(5) + f'(5) \geq 28$

$f'(5) - f'(2) \geq 12$

$f'(5) \geq 17$ ...(2)

13. Let α and β be the roots of $x^2-3x+p=0$ and γ and δ be the roots of $x^2-6x+q=0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression.Then ratio (2q+p): (2q-p) is:

(1) 33 :31 (2) 9 : 7 (3) 3 : 1 (4) 5 : 3

Sol. **(2)**

$x^2 - 3x + p = 0\ (\alpha, \beta)$

$x^2 - 6x + q = 0\ (\gamma, \delta)$

$\alpha + \beta = 3$

$\gamma + \delta = 6$

$\alpha = a,\ \beta = ar,\ \gamma = ar^2,\ \delta = ar^3$

$a(1 + r) = 3$ ...(1)

$ar^2(1 + r) = 6$ ...(2)
Divide (2) by (1)

$r^2 = 2, r = \sqrt{2} \Rightarrow a = \frac{3}{\sqrt{2}+1}$

$\alpha = \frac{3}{\sqrt{2}+1}, \beta = \frac{3\sqrt{2}}{\sqrt{2}+1}, \gamma = \frac{3.2}{\sqrt{2}+1}, \delta = \frac{3.2\sqrt{2}}{\sqrt{2}+1}$

$\alpha\beta = p = \frac{9\sqrt{2}}{\left(\sqrt{2}+1\right)^2}, \gamma\delta = \frac{36\sqrt{2}}{\left(\sqrt{2}+1\right)^2} \Rightarrow \frac{72+9}{72-9} = \frac{81}{63}$

= 9/7

14. Let $u = \frac{2z+i}{z-ki}$, z = x +iy and k>0. If the curve represented by Re(u) +Im(u) =1 intersects the y-axis at the points P and Q where PQ =5, then the value of k is :
(1) 4 (2) 1/2 (3) 2 (4) 3/2

Sol. **(3)**

$u = \frac{2z+i}{z-ki}$, z = x + iy

$= \frac{2x + i(2y+1)}{x + i(y-k)} \times \frac{x - i(y-k)}{x - i(y-k)}$

$\Rightarrow \frac{2x^2 + (2y+1)(y-k) + i\{2xy + x - 2xy + 2xk\}}{x^2 + (y-k)^2}$

Re(u) + Img(u) = 1
$2x^2 + (2y+1)(y - k) + x + 2xk = x^2 + (y - k)^2$
at y - axis, x = 0
$(2y + 1)(y - k) = (y - k)^2$
$2y^2 + y - 2yk - k = y^2 + k^2 - 2yk$
$y^2 + y - (k + k^2) = 0 \ (y_1, y_2)$
diff. of roots = 5

$\sqrt{1+4k+4k^2} = 5$
$4k^2 + 4k = 24$
$k^2 + k - 6 = 0$
$(k + 3)(k - 2) = 0$
k = 2

15. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?

(1) $a^2-d^2=0$ (B) $a^2-c^2=1$ (C) $0 \le a^2 + b^2 \le 1$ (D) $a^2 - b^2 = \frac{1}{2}$

Sol. **(4)**

$$\begin{bmatrix} c & is \\ is & c \end{bmatrix}\begin{bmatrix} c & is \\ is & c \end{bmatrix} = z\begin{bmatrix} c^2 - s^2 & 2ics \\ 2ics & c^2 - s^2 \end{bmatrix} = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix} \quad \text{(where } c = \cos\theta, s = \sin\theta))$$

$$A^5 = \begin{bmatrix} \cos(2^4\theta) & i\sin(2^4\theta) \\ i\sin(2^4\theta) & \cos(2^4\theta) \end{bmatrix}$$

$a = d = \cos(16\theta)$
$b = c = i\sin(16\theta)$
$a^2 - b^2 = \cos^2(16\theta) + \sin^2 16\theta = 1$

16. The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is:
(1) 3 (2) 9 (3) 7 (4) 5

Sol. **3**

$$\frac{5+7+10+12+14+15+x+y}{8} = 10$$

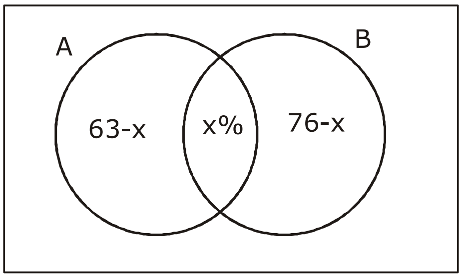
$x + y = 17$ ....(1)

$$\text{variance} = \frac{739 + x^2 + y^2}{8} - 100 = 13.5$$

$x^2 + y^2 = 169$ ...(2)
$\therefore x = 12, y = 5$
$|x - y| = 7$

17. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:
(1) 37 (2) 29 (3) 65 (4) 55

Sol. **(4)**



$A \cup B = 13 - x \le 100$
$x \ge 39$
also $x \le 63$

18. Given the following two statements:

$(S_1): (q \vee p) \rightarrow (P \leftrightarrow \sim q)$ is a tautology

$(S_2): \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then:

(1) only (S_1) is correct.
(2) both (S_1) and (S_2) are correct.
(3) only (S_2) is correct
(4) both (S_1) and (S_2) are not correct.

Sol. **(4)**

$S_1 =$

p	q	~ q	qvp	p ↔~ q	(qvp) → (p ↔~ q)
T	T	F	T	F	F
T	F	T	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T

S_1 is not correct

$S_2 =$

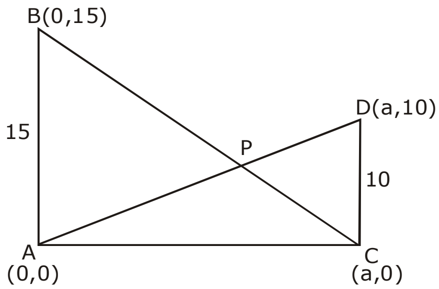
p	q	~ q	~ p	~ p ↔ q	~ q ∧ (~ p ↔ q)
T	T	F	F	F	F
T	F	T	F	T	T
F	T	F	T	T	F
F	F	T	T	F	F

S_2 is false

19. Two vertical poles AB=15 m and CD=10 m are standing apart on a horizontal ground with points A and C on the ground. If P is the point of intersection of BC and AD, then the height of P (in m) above the line AC is:

(1) 5 (2) 20/3 (3) 10/3 (4) 6

Sol. **(4)**



equation of AD : $y = \frac{10x}{a}$

equation of BC : $\frac{x}{a} + \frac{y}{15} = 1$

$\Rightarrow \frac{a.y}{10a} + \frac{y}{15} = 1 \Rightarrow \frac{3y+2y}{30} = 1$

$5y = 30 \Rightarrow y = 6$

20. If $\left(a+\sqrt{2}\ b\ \cos x\right)\left(a-\sqrt{2}\ b\cos y\right) = a^2 - b^2$, where a>b>0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ is:

(1) $\frac{a+b}{a-b}$ (2) $\frac{a-2b}{a+2b}$ (3) $\frac{a-b}{a+b}$ (4) $\frac{2a+b}{2a-b}$

Sol. **(1)**

$\left(a+\sqrt{2}\ b\ \cos x\right)\left(a-\sqrt{2}\ b\cos y\right) = a^2 - b^2$

diff both sides w.r.t y

$-\sqrt{2}\, b \sin x.\frac{dx}{dy}\left(a-\sqrt{2}b\cos y\right) + \left(a+\sqrt{2}b\cos x\right)\left(\sqrt{2}\, b \sin y\right) = 0$

$x = y = \frac{\pi}{4} \Rightarrow \frac{-bdx}{dy}(a-b)+(a+b)(b) = 0$

$\frac{dx}{dy} = \frac{a+b}{a-b}$

21. Suppose a differentiable function f(x) satisfies the identity $f(x+y)=f(x)+f(y)+xy^2+x^2y$, for all real x and y. If $\lim_{x\to 0}\frac{f(x)}{x} = 1$, then f'(3) is equal to............

Sol. $f(x + y) = f(x) + f(y) + xy^2 + x^2y$

$x = y = 0$

$f(0) = 2f(0) \Rightarrow f(0) = 0$

Partially diff. w.r.t. x
$f'(x + y) = f'(x) + y^2 + 2xy$
$x = 0$, $y = x$

$f'(x) = f'(0) + x^2$

given $\lim_{x \to 0} \frac{f(x)}{x} = 1$

$f'(x) = 1 + x^2$

by L' hospital

$\therefore f(x) = x + \frac{x^3}{3} + c$

$\lim_{x \to 0} \frac{f'(x)}{1} = 1$

put $x = 0 \Rightarrow c = 0$

$f'(0) = 1$

$f'(3) = 10$

22. If the equation of a plane P, passing through the intersection of the planes, x+4y-z+7=0 and 3x+y+5z=8 is ax+by+6z=15 for some a, b $\in$ R, then the distance of the point (3,2,-1) from the plane P is..........

Sol. $p_1 + \lambda p_2 = 0$
$(x + 4y - z + 7) + \lambda (3x + y + 5z - 8) = ax + by + 6z - 15$

$$\frac{1 - 3\lambda}{a} = \frac{4 + \lambda}{b} = \frac{-1 + 5\lambda}{6} = \frac{7 - 8\lambda}{-15}$$

$\therefore\ 15 - 75\lambda = 42 - 48\lambda$
$-27 = 27\lambda$
$\lambda = -1$
$\therefore$ plane is $-2x + 3y - 6z + 15 = 0$

$$d = \left|\frac{-6+6+6+15}{\sqrt{4+9+36}}\right| = 3$$

23. If the system of equations
x-2y+3z=9
2x+y+z=b
x-7y+az=24, has infinitely many solutions, then a-b is equal to.........

Sol. D = 0

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$1(a + 7) + 2(2a - 1) + 3(-14 - 1) = 0$
$a + 7 + 4a - 2 - 45 = 0$
$5a = 40$
$a = 8$

$$D_1 = \begin{vmatrix} 9 & -2 & 3 \\ b & 1 & 1 \\ 24 & -7 & 8 \end{vmatrix} = 0$$

$\Rightarrow 9(8 + 7) + 2(8b - 24) + 3(-7b - 24) = 0$
$\Rightarrow 135 + 16b - 48 - 21b - 72 = 0$
$15 = 5b \Rightarrow b = 3$
$a - b = 5$

24. Let $(2x^2+3x+4)^{10}=\sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ....

Sol. **8**

$(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$

a_7 = coeff of x^7
a_{13} = coeff of x^{13}

$\frac{10!}{p!q!r!}\left(2x^2\right)^p (3x)^q (4)^r$

for x^7

p	q	r
3	1	6
2	3	5
1	5	4
0	7	3

$a_7 = \frac{2^3.3.10!}{3!6!} + \frac{10!2^2.3^3}{2!3!5!} + \frac{10!2.3^5}{5!4!} + \frac{10!.3^7}{7!3!}$

for x^{13}

p	q	r
6	1	3
5	3	2
4	5	1
3	7	0

$a^{13} = \frac{2^6.3.10!}{6!3!} + \frac{2^5.3^3.10!}{5!3!2!} + \frac{2^4.3^5.10!}{4!5!} + \frac{2.^3 10!}{3!7!}$ $\therefore$ $\frac{a_7}{a_{13}} = 8$

25. The probability of a man hitting a target is $\frac{1}{10}$. The least number of shots required, so that the probability of his hitting the target at least once is greater than $\frac{1}{4}$, is ....

Sol. **3**

$P(H) = \frac{1}{10}$; $P(M) = \frac{9}{10}$

$P(H) + P(M).\ P(H) + P(M).\ P(M).\ P(H) + \ldots\ldots\ldots\ldots\ldots\ldots$

$= 1 - P(M)^n \geq \frac{1}{4}$

$= 1 - \left(\frac{9}{10}\right)^n \geq \frac{1}{4}$

$\left(\frac{9}{10}\right)^n \leq \frac{3}{4}$; $n \geq 3$