FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The temperature of equal masses of three different liquids x,y and z are 10ºC, 20ºC and 30ºC respectively. The temperature of mixture when x is mixed with y is 16ºC and that when y is mixed with z is 26ºC. The temperature of mixture when x and z are mixed will be :

(1) 28.32º C (2) 25.62º C

(3) 23.84ºC (4) 20.28ºC

Official Ans. by NTA (3)

Sol.

X	Y	Z
$m_1 = m$	$m_2 = m$	$m_3 = m$
$T_1 = 10°C$	$T_2 = 20°C$	$T_3 = 30°C$
s_1	s_2	s_3

when x & y are mixed, $T_{f_1} = 16°C$

$m_1s_1T + m_2s_2T_2 = (m_1s_1 + m_2s_2)Tf_1$

$s_1 \times 10 + s_2 \times 20 = (s_1 + s_2) \times 16$

$s_1 = \frac{2}{3}s_2$ ….(i)

when y & z are mixex, $T_{f_2} = 26°C$

$m_2s_2T + m_3s_3T_3 = (m_3s_3 + m_3s_3)Tf_2$

$s_2 \times 20 + s_3 \times 30 = (s_2 + s_3) \times 26$

$s_3 = \frac{3}{2}s_2$ .....(ii)

when x & z are mixex

$m_1s_1T_1 + m_3s_3T_3 = (m_1s_1 + m_3s_3)Tf$

$\frac{2}{3}s_2 \times 10 + \frac{2}{3}s_2 \times 20 = \left(\frac{2}{3}s_2 + \frac{3}{2}s_2\right)T_f$

$T_f = 23.84°C$

Ans (3)

2. The de-Broglie wavelength of a particle having kinetic energy E is λ. How much extra energy must be given to this particle so that the de-Broglie wavelength reduces to 75% of the initial value ?

(1) $\frac{1}{9}E$ (2) $\frac{7}{9}E$

(3) E (4) $\frac{16}{9}E$

Official Ans. by NTA (2)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$, $mv = \sqrt{2mE}$

$\lambda \propto \frac{1}{\sqrt{E}}$

$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{E_1}{E_2}} = \frac{3}{4}$, $\lambda_2 = 0.75\,\lambda_1$

$\frac{E_1}{E_2} = \left(\frac{3}{4}\right)^2$

$E_2 = \frac{16}{9}E_1 = \frac{16}{9}E$ $(E_1 = E)$

Extra energy given = $\frac{16}{9}E - E = \frac{7}{9}E$

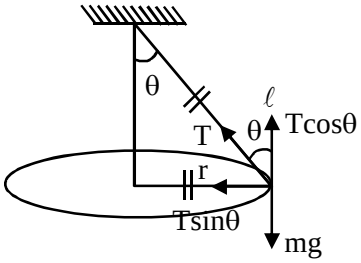
Ans. 2

3. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle will be :

(1) $\sqrt{rg}$ (2) $\sqrt{2rg}$ (3) $2\sqrt{rg}$ (4) $\sqrt{\frac{rg}{2}}$

Official Ans. by NTA (1)

Sol. Conical pendulum



$r = \frac{\ell}{\sqrt{2}}$

$\sin\theta = \frac{r}{\ell} = \frac{1}{\sqrt{2}}$

$\theta = 45°$

$T\sin\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$

Ans. 1

4. A cylindrical container of volume 4.0×10^{-3} m^3 contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400 K. The pressure of the mixture of gases is :

[Take gas constant as 8.3 J mol^{-1} K^{-1}]

(1) 249×10^1 Pa (2) 24.9×10^3 Pa

(3) 24.9×10^5 Pa (4) 24.9 Pa

Official Ans. by NTA (3)

Sol. $V = 4 \times 10^{-3}$ m^3

n = 3 moles

T = 400K

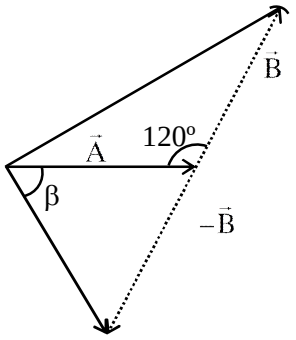
$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$P = \frac{3 \times 8.3 \times 400}{4 \times 10^{-3}}$$

$= 24.9 \times 10^5$ Pa

Ans 3

5. The angle between vector $(\vec{A})$ and $(\vec{A} - \vec{B})$ is :



(1) $\tan^{-1}\left(\frac{-\frac{B}{2}}{A - B\frac{\sqrt{3}}{2}}\right)$

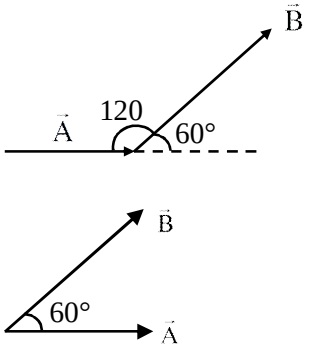
(2) $\tan^{-1}\left(\frac{A}{0.7B}\right)$

(3) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A - B}\right)$

(4) $\tan^{-1}\left(\frac{B\cos\theta}{A - B\sin\theta}\right)$

Official Ans. by NTA (3)

Sol.



Angle between $\vec{A}$ and $\vec{B}$, $\theta = 60°$

Angle betwenn $\vec{A}$ and $\vec{A} - \vec{B}$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$= \frac{B\sqrt{\frac{3}{2}}}{A - B \times \frac{1}{2}2}$$

$$\tan\alpha = \frac{\sqrt{3}B}{2A - B}$$

Ans 3

6. A light beam is described by $E = 800 \sin\omega\left(t - \frac{x}{c}\right)$ .An electron is allowed to move normal to the propagation of light beam with a speed of 3×10^7 ms^{-1}. What is the maximum magnetic force exerted on the electron ?

(1) 1.28×10^{-18} N

(2) 1.28×10^{-21} N

(3) 12.8×10^{-17} N

(4) 12.8×10^{-18} N

Official Ans. by NTA (4)

Sol. $\frac{E_0}{C} = B_0$

$$F_{max} = eB_0V$$

$$= 1.6 \times 10^{-19} \times \frac{800}{3 \times 10^8} \times 3 \times 10^7$$

$= 12.8 \times 10^{-18}$N

Ans. 4

7. The two thin coaxial rings, each of radius 'a' and having charges +Q and –Q respectively are separated by a distance of 's'. The potential difference between the centres of the two rings is :

(1) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$

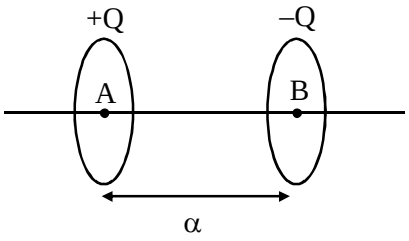
(2) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$

(3) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$

(4) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$

Official Ans. by NTA (4)

Sol.



$$V_A=\frac{KQ}{a}-\frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_B=\frac{-KQ}{a}+\frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_A-V_B=\frac{2KQ}{a}-\frac{2KQ}{\sqrt{a^2+s^2}}$$

$$=\frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{s^2+a^2}\right)$$

Ans 4

8. If you are provided a set of resistances 2Ω, 4Ω, 6Ω and 8Ω. Connect these resistances so as to obtain an equivalent resistance of $\frac{46}{3}\Omega$.

(1) 4Ω and 6Ω are in parallel with 2Ω and 8 Ω in series

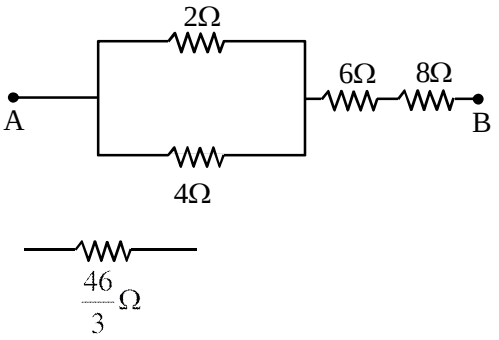
(2) 6Ω and 8Ω are in parallel with 2Ω and 4Ω in series

(3) 2Ω and 6Ω are in parallel with 4Ω and 8 Ω in series

(4) 2Ω and 4Ω are in parallel with 6Ω and 8Ω in series

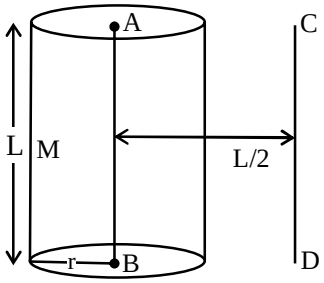
Official Ans. by NTA (4)

Sol.



Ans 4

9. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is 2.7 kg m^2.



(1) 14.9 kg / m^3 (2) 7.5 × 10^1 kg / m^3

(3) 7.5 × 10^2 kg/m^3 (4) 1.49 × 10^2 kg / m^3

Official Ans. by NTA (4)

Sol. Parallel axis theorem

$I = I_{CM} + Md^2$

$$I=\frac{Mr^2}{2}+M\left(\frac{L}{2}\right)^2$$

$$2.7=M\frac{(0.2)^2}{2}+M\left(\frac{0.8}{2}\right)^2$$

$$2.7=M\left[\frac{2}{100}+\frac{16}{100}\right]$$

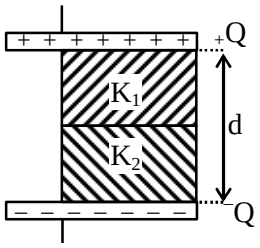
M = 15kg

$$\Rightarrow \rho=\frac{M}{\pi r^2 L}=\frac{15}{\pi(0.2)^2\times 0.8}$$

$= 0.1492 \times 10^3$

Ans. 4

10. A parallel - plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area A/2 and thickness d/2 are inserted in the space between the plates. The capacitance of the capacitor will be given by :



(1) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$

(2) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{2(K_1+K_2)}\right)$

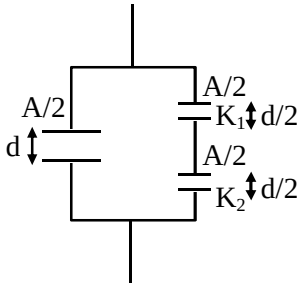
(3) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1+K_2}{K_1K_2}\right)$

(4) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{2(K_1+K_2)}{K_1K_2}\right)$

Official Ans. by NTA (1)

Sol. $C_{eq}=\frac{\frac{A}{2}\varepsilon_0}{d}+\frac{A\varepsilon_0}{d}\frac{K_1K_2}{K_1+K_2}$

$=\frac{A\varepsilon_0}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$



Ans.1

11. A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a :

(1) hyperbola

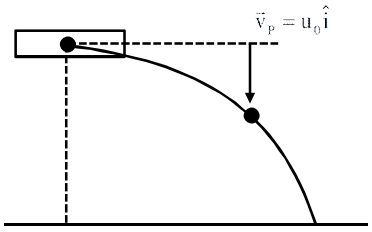
(2) parabola in the direction of motion of plane

(3) straight line vertically down the plane

(4) parabola in a direction opposite to the motion of plane

Official Ans. by NTA (3)

Sol.



$\vec{v}_B = u_0\hat{i} - gt\hat{j}$

$\vec{v}_{B/P} = \vec{v}_B - \vec{v}_P$

$\vec{v}_{B/P} = -8t\hat{j}$

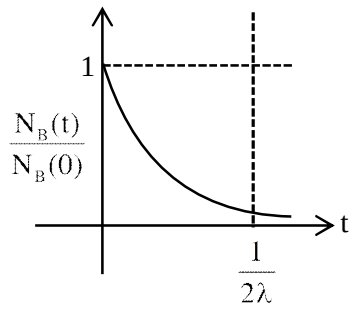
straight line vertically down

Ans.3

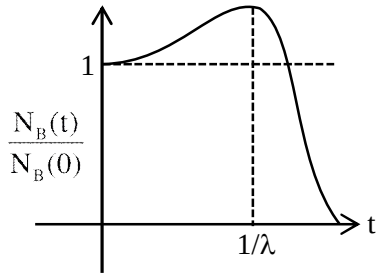
12. At time t = 0, a material is composed of two radioactive atoms A and B, where $N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is λ. However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t)$ / $N_B(0)$ with respect to time t ?

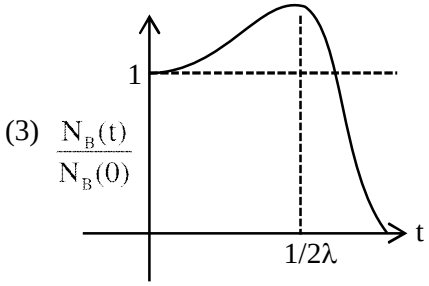
$\begin{bmatrix} N_A(0) = \text{No. of A atoms at } t=0 \\ N_B(0) = \text{No. of B atoms at } t=0 \end{bmatrix}$

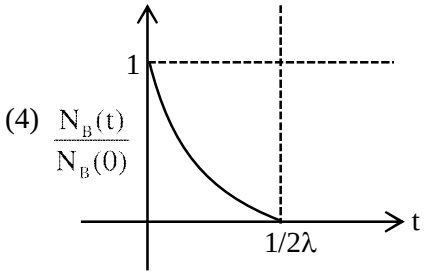
(1)



(2)



(3) 

(4) 

Official Ans. by NTA (3)

Sol.

$A \rightarrow B,\ B \rightarrow C$

$$\frac{dN_B}{dt} = \lambda N_A - \lambda N_B$$

$$\frac{dN_B}{dt} = 2\lambda N_{B_0} e^{-\lambda t} - \lambda N_B$$

$$e^{-\lambda t}\left(\frac{dN_B}{dt} + \lambda N_B\right) = 2\lambda N_{B_0} e^{-\lambda t} \times e^{\lambda t}$$

$$\frac{d}{dt}\left(N_B e^{\lambda t}\right) = 2\lambda N_{B_0}, \text{ on integrating}$$

$$N_B e^{\lambda t} = 2\lambda t N_{B_0} + N_{B_0}$$

$$N_B = N_{B_0}[1 + 2\lambda t]e^{-\lambda t}$$

$$\frac{dN_B}{dt} = 0 \text{ at } -\lambda[1 + 2\lambda t)e^{-\lambda t} + 2\lambda e^{-\lambda t} = 0$$

$$N_{B_{max}} \text{ at } t = \frac{1}{2\lambda}$$

13. A transmitting antenna at top of a tower has a height of 50 m and the height of receiving antenna is 80 m. What is range of communication for Line of Sight (LoS) mode ?

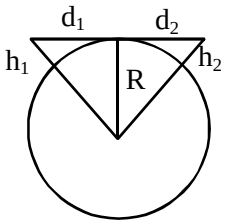
[use radius of earth = 6400 km]

(1) 45.5 km (2) 80.2 km

(3) 144.1 km (4) 57.28 km

Official Ans. by NTA (4)

Sol.



$$d_t = \sqrt{2Rh_1} + \sqrt{2Rh_2}$$

$$= \sqrt{2R}\left(\sqrt{h_1} + \sqrt{h_2}\right)$$

$$= (2 \times 6400 \times 10^3)^{1/2}\left(\sqrt{50} + \sqrt{80}\right)$$

= 3578 (7.07 + 8.94)

= 57.28 Km

14. A refrigerator consumes an average 35 W power to operate between temperature –10°C to 25°C. If there is no loss of energy then how much average heat per second does it transfer ?

(1) 263 J/s (2) 298 J/s (3) 350 J/s (4) 35 J/s

Official Ans. by NTA (1)

Sol. $$\frac{T_L}{T_H - T_L} = \text{C.O.P.} = \frac{\frac{dH}{dt}}{\frac{dW}{dt}}$$

$$\frac{263}{35} \times 35 = \frac{dH}{dt}$$

$$\frac{dH}{dt} = 263 \text{ watts}$$

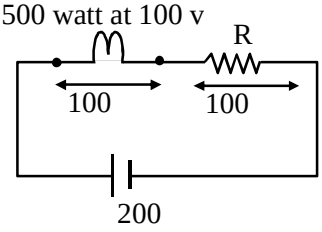
Ans.1

15. An electric bulb of 500 watt at 100 volt is used in a circuit having a 200 V supply. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500 W.

(1) 20 Ω (2) 30 Ω (3) 5 Ω (4) 10 Ω

Official Ans. by NTA (1)

Sol. 500 watt at 100 v



P = Vi

500 = Vi

i = 5 Amp

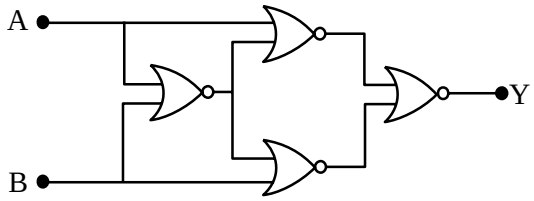
V = i × R

R = 20

Ans. 1

16. Four NOR gates are connected as shown in figure.

The truth table for the given figure is :



(1)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

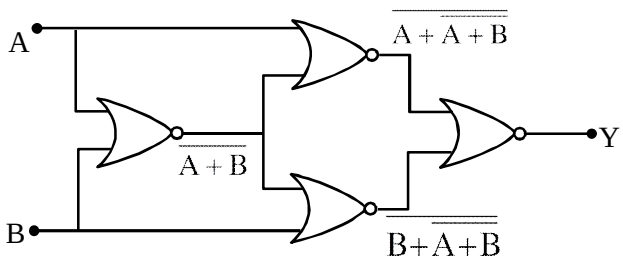
A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Official Ans. by NTA (4)

Sol.



$$y=\overline{(A+\overline{A+B})+(B+\overline{A+B})}$$

$$y=(A+\overline{A+B}).(B+\overline{A+B})$$

A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

Ans.4

17. Match List–I with List–II.

List-I		List-II	
(a)	Magnetic Induction	(i)	$ML^2T^{-2}A^{-1}$
(b)	Magnetic Flux	(ii)	$M^0L^{-1}A$
(c)	Magnetic Permeability	(iii)	$MT^{-2}A^{-1}$
(d)	Magnetization	(iv)	$MLT^{-2}A^{-2}$

Choose the most appropriate answer from the options given below :

(1) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Official Ans. by NTA (4)

Sol. (a) Magnetic Induction = $MT^{-2}A^{-1}$

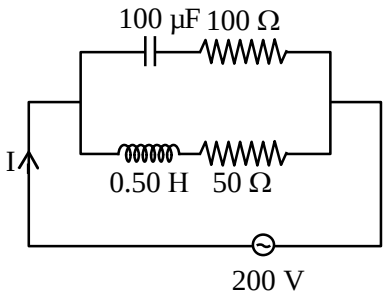
(b) Magnetic Flux = $ML^2T^{-2}A^{-1}$

(c) Magnetic Permeability = $MLT^{-2}A^{-2}$

(d) Magnetization = $M^0L^{-1}A$

Ans. 4

18. In the given circuit the AC source has ω = 100 rad s^{-1}. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?



(1) 5.9 A

(2) 4.24 A

(3) 0.94 A

(4) 6 A

Official Ans. by NTA (2)

Official Ans. by ALLEN (Bonus)

Sol. $Z_C = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

$= \sqrt{\left(\frac{1}{100 \times 100 \times 10^{-6}}\right)^2 + 100^2}$

$Z_C = \sqrt{(100)^2 + (100)^2}$

$= 100\sqrt{2}$

$Z_L = \sqrt{(\omega L)^2 + R^2}$

$\sqrt{(100 \times 0.5)^2 + 50^2}$

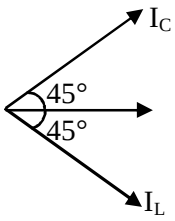
$= 50\sqrt{2}$

$i_C = \frac{200}{z_C} = \frac{200}{100\sqrt{2}} = \sqrt{2}$

$i_L = \frac{200}{z_L} = \frac{200}{50\sqrt{2}} = 2\sqrt{2}$

$\cos\phi_1 = \frac{100}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_1 = 45°$

$\cos\phi_2 = \frac{50}{50\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45°$



$I = \sqrt{I_C^2 + I_L^2}$

$= \sqrt{2+8}$

$= \sqrt{10}$

I = 3.16 A

Ans. 3.16

19. If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is:

(1) 86.4 s

(2) 4.32 s

(3) 43.2 s

(4) 8.64 s

Official Ans. by NTA (3)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$

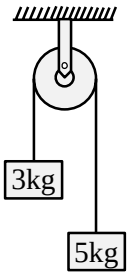
$\Delta T = \frac{1}{2} \times \frac{0.1}{100} \times 24 \times 3600$

$\Delta T = 43.2$

Ans. 3

20. Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2$ Nm^{-2}. What is the minimum radius of the wire?

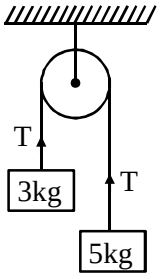
(Take g = 10 ms^{-2})



(1) 125 cm (2) 1250 cm

(3) 12.5 cm (4) 1.25 cm

Official Ans. by NTA (3)



Sol.

$T = \frac{2m_1m_2g}{m_1+m_2} = \frac{2 \times 3 \times 5 \times 10}{8}$

$= \frac{75}{2}$

Stress = $\frac{T}{A}$

$\frac{24}{\pi} \times 10^2 = \frac{75}{2 \times \pi R^2}$

$R^2 = \frac{75}{2 \times 24 \times 100} = \frac{3}{8 \times 24}$

$\Rightarrow$ R = 0.125 m

R = 12.5 cm

SECTION-B

1. Two waves are simultaneously passing through a string and their equations are :

$y_1 = A_1 \sin k(x–vt)$, $y_2 = A_2 \sin k(x–vt + x_0)$. Given amplitudes A_1 = 12 mm and A_2 = 5 mm, x_0 = 3.5 cm and wave number k = 6.28 cm^{-1}. The amplitude of resulting wave will be ..... mm.

Official Ans. by NTA (7)

Sol. $y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28 (x - vt)$

$y_2 = 5 \sin 6.28 (x - vt + 3.5)$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$= K(\Delta x)$

$$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$A_{net} = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$$

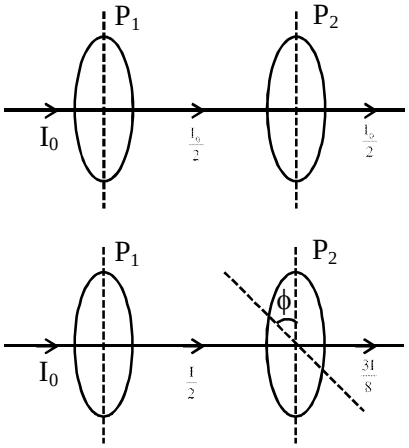
$$= \sqrt{144 + 25 - 120}$$

Ans. 7

2. A source of light is placed in front of a screen. Intensity of light on the screen is I. Two Polaroids P_1 and P_2 are so placed in between the source of light and screen that the intensity of light on screen is I/2. P_2 should be rotated by an angle of ..... (degrees) so that the intensity of light on the screen becomes $\frac{3I}{8}$.

Official Ans. by NTA (30)

Sol. $$I = \frac{I_0}{2}\cos^2\phi$$



$$\frac{I}{2}\cos^2\phi = \frac{3I}{8}$$

$$\cos^2\phi = \frac{3}{4}$$

$$\cos^2\phi = \frac{\sqrt{3}}{2}$$

$\Rightarrow \phi = 30$

Ans. 30

3. If the maximum value of accelerating potential provided by a ratio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is .....

[$m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C,

Speed of light = 3×10^8 m/s]

Official Ans. by NTA (543)

Sol. V = 12 kV

Number of revolution = n

$$n\left[2 \times q_P \times V\right] = \frac{1}{2}m_P \times v_P^2$$

$n[2\times 1.6 \times 10^{-19} \times 12 \times 10^3$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times \left[\frac{3\times 10^8}{6}\right]^2$$

$n(38.4\times10^{-16}) = 0.2087 \times 10^{-11}$

n = 543.4

Ans. 543

4. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum to known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as .....%

Official Ans. by NTA (14)

Sol. $$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \ell = \frac{T^2 g}{4\pi^2}$$

$$E = mg\ell\frac{\theta^2}{2} = mg^2\frac{T^2\theta^2}{8\pi^2}$$

$$\frac{dE}{E} = 2\left(\frac{dg}{g} + \frac{dT}{T}\right)$$

$= (4 + 3) = 14\%$

5. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of 3.0×10^{-2} T. The maximum emf induced the coil will be ..... $\times 10^{-2}$ volt (rounded off to the nearest integer)

Official Ans. by NTA (60)

Sol. Maximum emf $\varepsilon = N\omega AB$

$N = 20, \omega = 50, B = 3 \times 10^{-2}T$

$\varepsilon = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} = 60.28 \times 10^{-2}$

Rounded off to nearest integer = 60

Ans.60

6. Two simple harmonic motions are represented by the equations

$x_1 = 5 \sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}\,(\sin 2\pi t + \cos 2\pi t)$.

The amplitude of second motion is ..... times the amplitude in first motion.

Official Ans. by NTA (2)

Sol. $x_2 = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 2\pi t + \frac{1}{\sqrt{2}}\cos 2\pi t\right)\sqrt{2}$

$= 10\sin\left(2\pi t + \frac{\pi}{4}\right)$

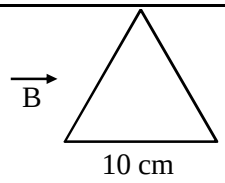
$\therefore \frac{A_2}{A_1} = \frac{10}{5} = 2$

Ans. 2

7. A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20 mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x} \times 10^{-5}$ Nm. The value of x is.....

Official Ans. by NTA (3)

Sol.



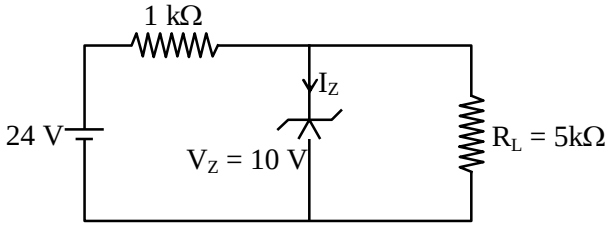
$\vec{\tau} = \vec{M} \times \vec{B} = MB\sin 90°$

$= MB = \frac{i\sqrt{3}\ell^2}{4}B$

$= \sqrt{3} \times 10^{-5}\,N - m$

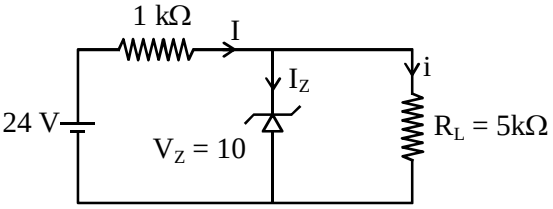
Ans. 3

8. For the given circuit, the power across zener diode is ..... mW.



Official Ans. by NTA (120)

Sol.



$i = \frac{10V}{5k\Omega} = 2mA$

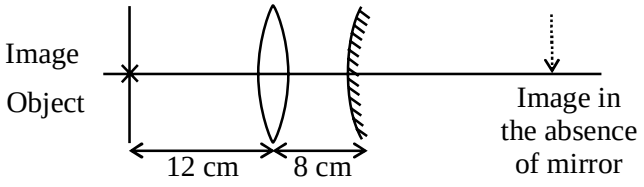
$I = \frac{14V}{1k\Omega} = 14mA$

$\therefore I_z = 12mA$

$\therefore P = I_z V_z = 120$ mW

Ans. 120

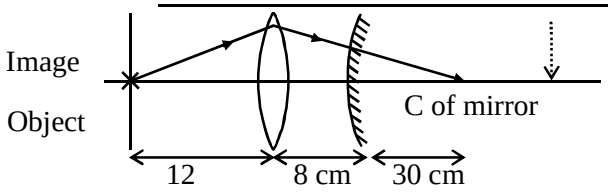
9. An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.



When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be .....(cm)

Official Ans. by NTA (50)

Sol.



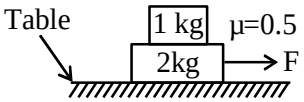
For the object to coincide with image, the light must fall perpendicularly to mirror. Which means that the light will have to converge at C of mirror.

Without the mirror also, the light would coverage at C.

So the distance is : 12 + 8 + 30 = 50 cm

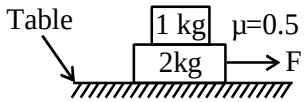
10. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is .....N.

(take g = 10 ms^{-2})

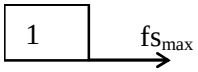


Official Ans. by NTA (15)

Sol.



F = 3a (For system) ….(i)



$fs_{max} = 1a$ (for 1kg block) ….(ii)

$\mu \times 1 \times g = a$

$\Rightarrow 5 = a$

F = 15N

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

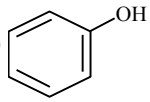
(Held On Thursday 26th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

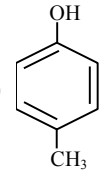
CHEMISTRY

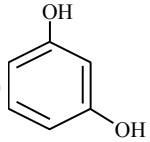
TEST PAPER WITH SOLUTION

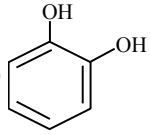
SECTION-A

1. Which one of the following phenols does not give colour when condensed with phthalic anhydride in presence of conc. H_2SO_4 ?

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. Only p-methyl, phenol does not give any colour with phthalic anhydroxide with cons. H_2SO_4.

2. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Photochemical smog causes cracking of rubber.

Reason (R) : Presence of ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate in photochemical smog makes it oxidizing.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**

Official Ans. by NTA (4)

Sol. Photochemical smog causes cracking of rubber, the common component of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyle nitrate (PAN).

3. The interaction energy of London forces between two particles is proportional to r^x, where r is the distance between the particles. The value of x is :

(1) 3 (2) –3

(3) –6 (4) 6

Official Ans. by NTA (3)

Sol. For london dispersion forces.

$$E \propto \frac{1}{r^6}$$

Hence x = –6

4. The number of non-ionisable hydrogen atoms present in the final product obtained from the hydrolysis of PCl_5 is :

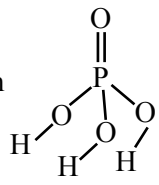
(1) 0 (2) 2 (3) 1 (4) 3

Official Ans. by NTA (1)

Sol. $PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$

$\downarrow + 3H_2O$

$H_3PO_4 + 3HCl$

In  + HCl

all hydrogens are ionisable

$\therefore$ Ans is zero.

5. The bond order and magnetic behaviour of O_2^- ion are, respectively :

(1) 1.5 and paramagnetic

(2) 1.5 and diamagnetic

(3) 2 and diamagnetic

(4) 1 and paramagnetic

Official Ans. by NTA (1)

Sol. $O_2^- = (\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2$

$\left(\pi_{2p_x}^2 = \pi_{2p_y}^2\right)\left(\pi_{2p_x}^{*2} = \pi_{2p_y}^{*1}\right)$

Bond order $= \frac{10-7}{2} = 1.5$

and paramagnetic

6. Given below are two statements : one is labelled as **Assertion (A)** and other is labelled as **Reason (R)**.

Assertion (A) : Sucrose is a disaccharide and a non-reducing sugar.

Reason (R) : Sucrose involves glycosidic linkage between C_1 of β-glucose and C_2 of α-fructose.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**

Official Ans. by NTA (3)

Sol. Surcrose is example of disaccharide & non reducing sugar

Assertion : correct

Sucrose involves glycosidic linkage between C_1 of α-D-glucose C_2 of β-D-fructose

Reason : Incorrect

7. Match **List-I** with **List-II** :

List-I (Chemical Reaction)	**List-II (Reagent used)**
(a) $CH_3COOCH_2CH_3 \rightarrow CH_3CH_2OH$	(i) CH_3MgBr / H_3O^+ (1.equivalent)
(b) $CH_3COOCH_3 \rightarrow CH_3CHO$	(ii) H_2SO_4 / H_2O
(c) $CH_3C \equiv N \rightarrow CH_3CHO$	(iii) DIBAL-H/H_2O
(d) $CH_3C \equiv N \rightarrow CH_3{-}\overset{\overset{O}{\|}}{C}{-}CH_3$	(iv) $SnCl_2$, HCl/H_2O

Choose the most appropriate match :

(1) a-ii, b-iv, c-iii, d-i

(2) a-iv, b-ii, c-iii, d-i

(3) a-ii, b-iii, c-iv, d-i

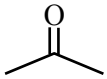
(4) a-iii, b-ii, c-i, d-iv

Official Ans. by NTA (3)

Sol. $CH_3{-}\overset{\overset{O}{\|}}{C}{-}O{-}CH_2CH_3 \xrightarrow{H_3O^+} CH_3CO_2H + CH_3CH_2OH$

$CH_3{-}\overset{\overset{O}{\|}}{C}{-}O{-}CH_3 \xrightarrow[-78^\circ C]{DIBALH/H_2O} CH_3CHO$

$CH_3{-}CN \xrightarrow[/H_2O]{SnCl_2+HCl} CH_3CH{=}O$

$CH_3{-}C{\equiv}N \xrightarrow[/H_3O^+]{CH_3MgBr\ (1eq)}$ 

8. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Barium carbonate is insoluble in water and is highly stable.

Reason (R) : The thermal stability of the carbonates increases with increasing cationic size.

(1) Both **(A)** and **(R)** are true but **(R)** is the true explanation of **(A)**

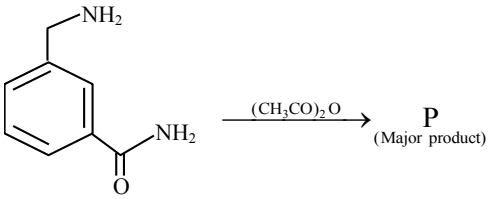
(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is not the true explanation of **(A)**

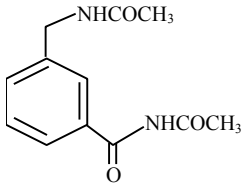
(4) **(A)** is false but **(R)** is true.

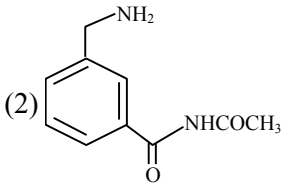
Official Ans. by NTA (1)

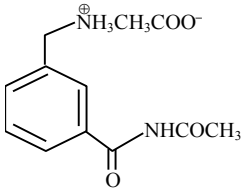
Sol. In IIA group on moving down the group size of cation increases and show thermal stability of carbonate increases.

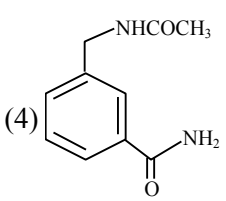
9. 

The major product in the above reaction is :

(1) 

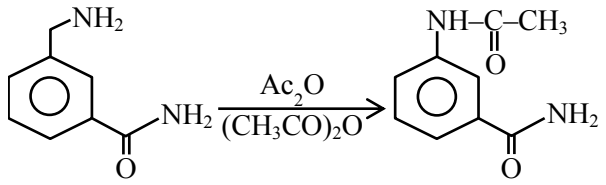
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



10. Indicate the complex/complex ion which did not show any geometrical isomerism :

(1) $[CoCl_2(en)_2]$ (2) $[Co(CN)_5(NC)]^{3-}$

(3) $[Co(NH_3)_3(NO_2)_3]$ (4) $[Co(NH_3)_4Cl_2]^+$

Official Ans. by NTA (2)

Sol. (1) $[CoCl_2(en)_2]$ show
Cis-trans isomerism

(2) $[Co(CN)_5(NC)]^{-3}$ can't
Show G.I.

(3) $[Co(NH_3)_3(NO_2)_3]$
Show fac & mer isomerism

(4) $[Co(NH_3)_4Cl_2]^{\oplus}$ show cis & trans isomerism

11. The sol given below with negatively charged colloidal particles is :

(1) $FeCl_3$ added to hot water

(2) KI added to $AgNO_3$ solution

(3) $AgNO_3$ added to KI solution

(4) $Al_2O_3.xH_2O$ in water

Official Ans. by NTA (3)

Sol.

12. Given below are two statements :

Statement I : Sphalerite is a sulphide ore of zinc and copper glance is a sulphide ore of copper.

Statement II : It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants' in a froth flotation method.

Choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false.

(2) Both **Statement I** and **Statement II** are true.

(3) **Statement I** is false but **Statement II** is true.

(4) Both **Statement I** and **Statement II** are false.

Official Ans. by NTA (2)

Sol. Sphalerite-ZnS, copper glance - Cu_2S two sulphide ores can be separated by adjusting proportions of oil to water or by using ' Depressants '

13. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Heavy water is used for the study of reaction mechanism.

Reason (R) : The rate of reaction for the cleavage of O - H bond is slower than that of O-D bond.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**.

(2) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**.

(3) **(A)** is false but **(R)** is true.

(4) **(A)** is true but **(R)** is false.

Official Ans. by NTA (4)

Sol. D_2O in used for the study of reaction mechanism.
Rate of reaction for the cleavage of O–H bond > O-D bond.

14. Arrange the following Cobalt complexes in the order of increasing Crystal Field Stabilization Energy (CFSE) value.

Complexes : $\underset{A}{[CoF_6]^{3-}}, \underset{B}{[Co(H_2O)_6]^{2+}}, \underset{C}{[Co(NH_3)_6]^{3+}}$ and $[Co(en)_3]^{3+}$

Choose the **correct** option :

(1) A < B < C < D (2) B < A < C < D

(3) B < C < D < A (4) C < D < B < A

Official Ans. by NTA (2)

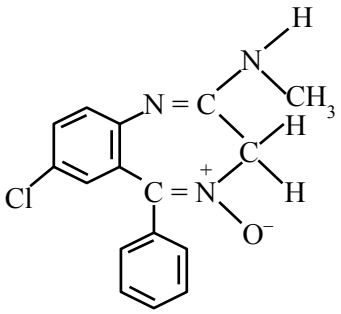
Sol. (i) CFSE $\propto$ charge or oxidation no. of central metal ion.

(ii) CFSE $\propto$ strength of ligand
en > NH_3 > H_2O > F^-

$\therefore$ order of CFSE

$$\overset{III}{[Co(en)_3]^{+3}} > \overset{III}{Co(NH_3)_6]^{+3}} > \overset{III}{[CoF_6]^{-3}} > \overset{II}{[Co(H_2O)_6]^{+2}}$$

15.



Chlordiazepoxide

The class of drug to which chlordiazepoxide with above structure belongs is :

(1) Antacid (2) Analgesic

(3) Tranquilizer (4) Antibiotic

Official Ans. by NTA (3)

Sol. The drug named chlordiate poxide is example of tranquilizer.

16. Chalcogen group elements are :

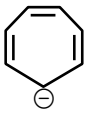
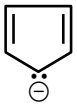
(1) Se, Tb and Pu. (2) Se, Te and Po.

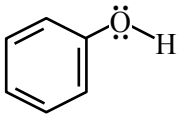
(3) S, Te and Pm. (4) O, Ti and Po.

Official Ans. by NTA (2)

Sol. Group 16/oxygen family is known as Chalcogens the members are O, S, Se, Te, Po

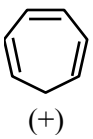
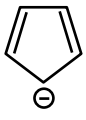
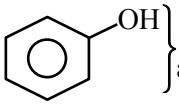
17. Which one of the following compounds is not aromatic ?

(1)  (2) 

(3)  (4) 

Official Ans. by NTA (3)

Sol.  : Non aromatic

 ;  ;  All are aromatic

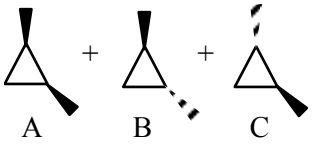
(+)

18. The number of stereoisomers possible for 1,2-dimethyl cyclopropane is :

(1) One (2) Four

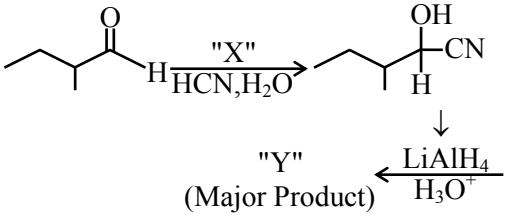
(3) Two (4) Three

Official Ans. by NTA (4)

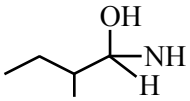
Sol. 

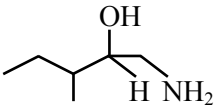
A B C

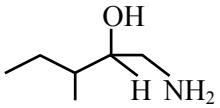
19.

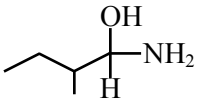


Consider the given reaction, Identify **'X'** and **'Y'** :

(1) X – NaOH Y – 

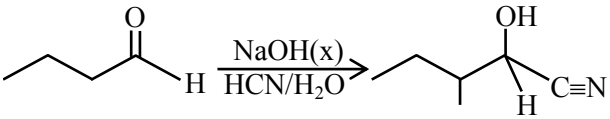
(2) X – HNO_3 Y – 

(3) X – NaOH Y – 

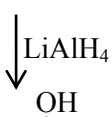
(4) X – HNO_3 Y – 

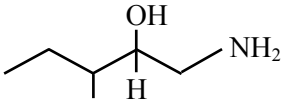
Official Ans. by NTA (3)

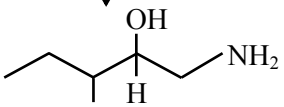
Sol.



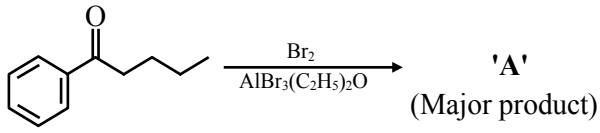
X ⇒ NaOH



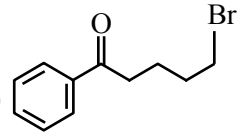
Y ⇒ 

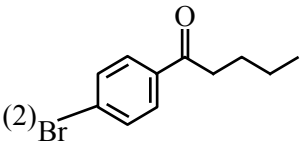


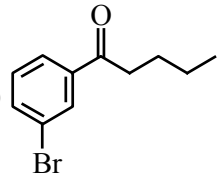
γ(major product)

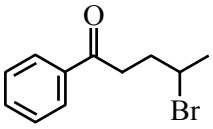
20. 

Consider the given reaction, the product **A** is :

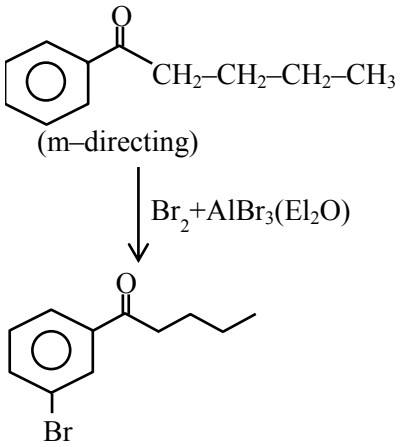
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

SECTION-B

1. In the sulphur estimation, 0.471 g of an organic compound gave 1.44 g of barium sulphate. The percentage of sulphur in the compound is _____%. (Nearest integer)

(Atomic Mass of Ba = 137 u)

Official Ans. by NTA (42)

Sol. Molecular mass of $BaSO_4$ = 233 g

$\because$ 233 $BaSO_4$ contain $\rightarrow$ 32 g sulphur

$\therefore$ 1.44 g $BaSO_4$ contain $\rightarrow \dfrac{32}{233} \times 1.44$ g sulphur

given : 0.471 g of organic compound

$$\% \text{ of S} = \frac{32 \times 1.44}{233 \times 0.471} \times 100 = 41.98\% \approx 42\%$$

OR

$$\underset{W_{OC}=0.471\,g}{\boxed{O.C.}} \longrightarrow \underset{1.44\,g}{BaSO_4}$$

$$\Rightarrow n_s = n_{BaSO_4} = \frac{1.44}{233}$$

$$\Rightarrow w_s = \frac{1.44}{233} \times 32g$$

$$\text{therefore } \%S = \frac{W_s}{W_{O.C.}} \times 100 = \frac{1.44 \times 32}{233 \times 0.471} \times 100$$

$$= \frac{46.08}{109.743} \times 100 = 41.98 \simeq 42$$

2. The equilibrium constant K_c at 298 K for the reaction $A + B \rightleftharpoons C + D$ is 100. Starting with an equimolar solution with concentrations of A, B, C and D all equal to 1M, the equilibrium concentration of D is_____ $\times 10^{-2}$ M. (Nearest integer)

Official Ans. by NTA (182)

Sol. $A + B \rightleftharpoons C + D : K_{eq} = 100$

1M 1M 1M 1M

First check direction of reversible reaction.

Since $Q_C = \dfrac{[C][D]}{[A][B]} = 1 < K_{eq.} \Rightarrow$ reaction will move in forward direction to attain equilibrium state.

$\Rightarrow A + B \rightleftharpoons C + D : K_{eq} = 100$

to 1 1 1 1

$t_{eq.}$ 1–x 1–x 1+x 1+x

$$\text{Now : } K_{eq} = 100 = \frac{(1+x)(1+x)}{(1-x)(1-x)}$$

$$\Rightarrow \boxed{100 = \left(\frac{1+x}{1-x}\right)^2}$$

(i) $10 = \left(\dfrac{1+x}{1-x}\right)$

$\Rightarrow 10 - 10x = 1+x$

$\Rightarrow 11x = 9$

$\Rightarrow \boxed{x = \dfrac{9}{11}}$

(ii) $-10 = \dfrac{1+x}{1-x}$

$\Rightarrow -10 + 10x = 1 + x$

$\Rightarrow -9x = -11$

$\Rightarrow \boxed{x = \frac{11}{9}}$

$\rightarrow$ 'x' cannot be more than one, therefore not valid.

therefore equation concretion of (D) = 1 + x

$$= 1 + \frac{9}{11} = \frac{20}{11}$$

$$= 1.8181 = 181.81 \times 10^{-2}$$

$$\simeq 182 \times 10^{-2}$$

3. For water $\Delta_{vap} H = 41$ kJ mol^{-1} at 373 K and 1 bar pressure. Assuming that water vapour is an ideal gas that occupies a much larger volume than liquid water, the internal energy change during evaporation of water is_____kJ mol^{-1}

[Use : R = 8.3 J mol^{-1} K^{-1}]

Official Ans. by NTA (38)

Sol. Given equation is

$$H_2O(\ell) \longrightarrow H_2O(g) : \Delta H = 41 \ \frac{kJ}{mol}$$

$\Rightarrow$ From the relation : $\Delta H = \Delta U + \Delta n_g RT$

$$\Rightarrow 41 \frac{kJ}{mol} = \Delta U + (1) \times \frac{8.3}{1000} \times 373$$

$\Rightarrow$ DU = 41 – 3.0959

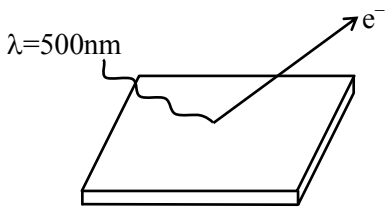
= 38 kJ/mol

4. A metal surface is exposed to 500 nm radiation. The threshold frequency of the metal for photoelectric current is 4.3×10^{14} Hz. The velocity of ejected electron is _____$\times 10^5$ ms^{-1} (Nearest integer)

[Use : h = 6.63×10^{-34} Js, $m_e = 9.0 \times 10^{-31}$ kg]

Official Ans. by NTA (5)

Sol.



υ : speed of electron having max. K.E.

$\Rightarrow$ from Einstein equation : $E = \phi + K.E._{max}$

$$\Rightarrow \frac{hc}{\lambda} = h\upsilon_0 + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}} = 6.63 \times 10^{-34} \times 4.3 \times 10^{14} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{6.63 \times 30 \times 10^{-20}}{5} = 6.63 \times 4.3 \times 10^{-20} + \frac{1}{2}mv^2$$

$$\Rightarrow 11.271 \times 10^{-20} \text{ J} = \frac{1}{2} \times 9 \times 10^{-31} \times \upsilon^2$$

$$\Rightarrow \boxed{\upsilon = 5 \times 10^5 \text{ m/sec.}}$$

5. For the galvanic cell,

$Zn(s) + Cu^{2+}$ (0.02 M) $\rightarrow Zn^{2+}$ (0.04 M) + Cu(s),

E_{cell} =_____ $\times 10^{-2}$ V. (Nearest integer)

[Use : $E^0_{Cu/Cu^{2+}} = -0.34$ V, $E^0_{Zn/Zn^{2+}} = +0.76$ V, $\frac{2.303\ RT}{F} = 0.059$ V]

Official Ans. by NTA (109)

Sol. Galvanic cell:

$$Zn_{(s)} + \underset{0.02M}{Cu^{+2}_{(aq.)}} \rightarrow \underset{0.04M}{Zn^{+2}} + Cu(s)$$

Nernst equation = $F_{cell} = E^o_{cell} - \frac{0.059}{2} \log \frac{[2n^{+2}]}{[Cu^{+2}]}$

$$\Rightarrow E_{cell}\left[E^o_{cell} - E^o_{Zn^{+2}/Zn}\right] - \frac{0.059}{2} \log \frac{0.04}{0.02}$$

$$\Rightarrow E_{cell}\left[0.34 - (-0.76)\right] - \frac{0.059}{2} \log^2$$

$$\Rightarrow E_{cell} 1 - 1 - \frac{0.059}{2} \times 0.3010$$

$$= 1.0911 = 109.11 \times 10^{-2}$$

$$= 109$$

6. 100 mL of Na_3PO_4 solution contains 3.45 g of sodium. The molarity of the solution is _____$\times 10^{-2}$ mol L^{-1} . (Nearest integer)

[Atomic Masses - Na : 23.0 u, O : 16.0 u, P : 31.0 u]

Official Ans. by NTA (50)

Sol.

$\boxed{Na_3PO_4}$ 100ml Sol.

$Na_3PO_4 \longrightarrow 3Na$

$\frac{1}{3} \times \frac{3.45}{23}$ mol

3.45g

$\frac{3.45}{23}$ mol

therefore molarity of Na_3PO_4 Solution =

$$\frac{n_{Na_3PO_4}}{\text{volume of solution in L}}$$

$$= \frac{\frac{1}{3}\times\frac{3.45}{23}\text{mol}}{0.1\,\text{L}}$$

$$= 0.5 = 50 \times 10^{-2}$$

7. The overall stability constant of the complex ion $[Cu(NH_3)_4]^{2+}$ is 2.1×10^{13}. The overall dissociations constant is $y \times 10^{-14}$. Then y is ____.(Nearest integer)

Official Ans. by NTA (5)

Sol. Given $k_f = 2.1 \times 10^{13}$

$$K_d = \frac{1}{k_f} = 4.7\times10^{-14}$$

$$\therefore y = 4.7 \approx 5$$

8. 83 g of ethylene glycol dissolved in 625 g of water. The freezing point of the solution is _____K. (Nearest integer)

[Use : Molal Freezing point depression constant of water = 1.86 K kg mol^{-1}]

Freezing Point of water = 273 K

Atomic masses : C : 12.0 u, O : 16.0 u, H : 1.0 u]

Official Ans. by NTA (269)

Sol. $k_f = 1.86$ k. kg/mol

$T_f^o = 273$ k

solvent : H_2O(625 g)

Solute : 83 g $\begin{pmatrix} CH_2-CH_2 \\ | \quad | \\ OH \quad OH \end{pmatrix}$ $\Rightarrow$ Non dissociative solute

$$\Rightarrow \Delta T_f = k_f \times m$$

$$\Rightarrow \left(T_f^o - T_f^1\right) = 1.86\times\frac{83/62}{624/1000}$$

$$\Rightarrow 273 - T_f^1 = \frac{1.86\times83\times1000}{62\times625} = \frac{154380}{38750}$$

$$\Rightarrow 273 - T_f^1 = 4$$

$$\Rightarrow \boxed{T_f^1 = 259\ \text{K}}$$

9. The reaction rate for the reaction

$$[PtCl_4]^{2-} + H_2O \rightleftharpoons [Pt(H_2O)Cl_3]^- + Cl^-$$

was measured as a function of concentrations of different species. It was observed that

$$\frac{-d\left[\left[PtCl_4\right]^{2-}\right]}{dt} = 4.8 \times 10^{-5}\left[\left[PtCl_4\right]^{2-}\right] - 2.4 \times 10^{-3}\left[\left[Pt(H_2O)Cl_3\right]^-\right]\left[Cl^-\right].$$

where square brackets are used to denote molar concentrations. The equilibrium constant K_c=_____. (Nearest integer)

Official Ans. by NTA (50)

Sol. $[PtCl_4]^{-2} + H_2O \rightleftharpoons [Pt(H_2O)Cl_3]^- + Cl^-$

$$\frac{-d[Pt\ Cl_4]^{-2}}{dt} = 4.8 \times 10^{-5}\,[PtCl_4^{-2}] - 2.4 \times 10^3\,[Pt(H_2O)Cl_3][\overset{o}{u}]$$

$$\Rightarrow K_{eq} = \frac{k_f}{k_b} = \frac{4.8\times10^{-5}}{2.4\times10^{-3}} = 0.02$$

10. A chloro compound "A".

(i) forms aldehydes on ozonolysis followed by the hydrolysis.

(ii) when vaporized completely 1.53 g of A, gives 448 mL of vapour at STP.

The number of carbon atoms in a molecule of compound **A** is _____.

Official Ans. by NTA (3)

Sol. 448 ml of A $\Rightarrow$ 1.53 gm A

22400 ml of A $\Rightarrow \frac{1.53}{445} \times 22400$ gm A = 7650

$$H_3CHC{-}CH{-}Cl \xrightarrow[Zn/H_2O]{O_3} \underset{\text{Aldehyde}}{CH_3{-}CH{=}O}$$

It has 3 carbon atoms

& mm is 36 + 5 + 35.5 = 76.5

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let [t] denote the greatest integer less than or equal to t. Let f(x) = x – [x], g(x) = 1 – x + [x], and h(x) = min{f(x), g(x)}, x ∈ [–2, 2]. Then h is :

(1) continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

(2) not continuous at exactly three points in [–2, 2]

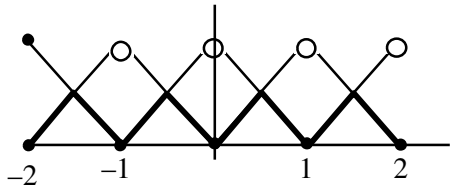
(3) continuous in [–2, 2] but not differentiable at exactly three points in (–2, 2)

(4) not continuous at exactly four points in [–2, 2]

Official Ans. by NTA (1)

Sol. $\min\{x - [x], 1 - x + [x]\}$

$h(x) = \min\{x - [x], 1 - [x - [x]]\}$



$\Rightarrow$ always continuous in [–2, 2]

but non differentiable at 7 Points

2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to :

(1) $A^6 - A$ (2) A^5

(3) $A^5 - A$ (4) A^6

Official Ans. by NTA (1)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^{2025} - A^{2020} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^6 - A = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. The local maximum value of the function

$f(x) = \left(\frac{2}{x}\right)^{x^2}, x > 0,$ is

(1) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$ (2) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(3) $(e)^{\frac{2}{e}}$ (4) 1

Official Ans. by NTA (3)

Sol. $f(x) = \left(\frac{2}{x}\right)^{x^2}$; $x > 0$

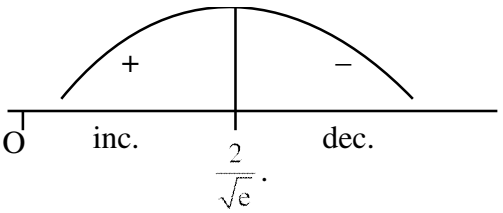
$\ell n\, f(x) = x^2 (\ell n 2 - \ell n x)$

$f'(x) = f(x) \{-x + (\ell n\, 2 - \ell n\, x)2x\}$

$f'(x) = \underbrace{f(x)}_{+} . \underbrace{x}_{+} \underbrace{(2\ell n 2 - 2\ell n x - 1)}_{g(x)}$

$g(x) = 2\ell\, n^2 - 2\ell\, n\, x - 1$

$= \ell n \frac{4}{x^2} - 1 = 0 \Rightarrow x = \frac{2}{\sqrt{e}}$



$LM = \frac{2}{\sqrt{e}}$

Local maximum value $= \left(\frac{2}{2/\sqrt{e}}\right)^{\frac{4}{e}} \Rightarrow e^{\frac{2}{e}}$

4. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbf{R}$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to :

(1) 100 (2) 25

(3) 16 (4) 36

Official Ans. by NTA (2)

Sol. $I = \int_0^5 \frac{x+[x]}{e^{x-[x]}}dx$

$$\int_0^1 \frac{x}{e^x}dx + \int_1^2 \frac{x+1}{e^{x-1}}dx + \int_2^3 \frac{x+2}{e^{x-2}}dx + ...\int_4^5 \frac{x+4}{e^{x-4}}dx$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$x = t+1 \qquad x = z+2 \qquad x = y+4$$

$$\int_0^1 \frac{t+2}{e^t}dt + \int_0^1 \frac{z+4}{e^z}dz + ... + \int_0^1 \frac{y+8}{e^y}dx$$

$$\Rightarrow \int_0^5 \frac{5x+20}{e^x}dt = 5\int_0^1 \frac{x+4}{e^x}dx$$

$$\Rightarrow 5\int_0^1 (x+4)e^{-x}dx$$

$$\Rightarrow 5e^{-x}(-x-5)|_0^1 \Rightarrow -\frac{30}{e} + 25$$

$\alpha = -30$

$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$

$(\alpha + \beta)^2 = 5^2 = 25$

5. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to :

(1) $4\sqrt{3}$ (2) 6

(3) $6\sqrt{3}$ (4) $3\sqrt{6}$

Official Ans. by NTA (3)

Sol. $P(-2\sqrt{6}, \sqrt{3})$ lies on hyperbola

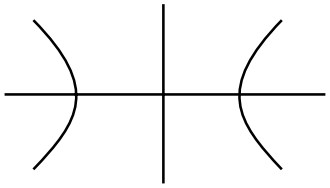
$$\Rightarrow \frac{24}{a^2} - \frac{3}{b^2} = 1 \quad .....(i)$$

$$e = \frac{\sqrt{5}}{2} \Rightarrow b^2 = a^2\left(\frac{5}{4} - 1\right) \Rightarrow 4b^2 = a^2$$

Put in (i) $\Rightarrow \frac{6}{b^2} - \frac{3}{b^2} = 1 \Rightarrow b = \sqrt{3}$

$\Rightarrow a = \sqrt{12}$

$\frac{x^2}{12} - \frac{y^2}{3} = 1$



Tangent at P :

$$\frac{-x}{\sqrt{6}} - \frac{y}{\sqrt{3}} = 1 \Rightarrow Q\ (0, \sqrt{3})$$

Slope of $T = -\frac{1}{\sqrt{2}}$

Normal at P :

$$y - \sqrt{3} = \sqrt{2}(x + 2\sqrt{6})$$

$\Rightarrow R = (0, 5\sqrt{3})$

$QR = 6\sqrt{3}$

6. Let y(x) be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0, x > 0$. If $y(e) = 1$, then $y(1)$ is equal to :

(1) 0 (2) 2

(3) $\log_e 2$ (4) $\log_e(2e)$

Official Ans. by NTA (3)

Sol. $2x^2dy + (e^y - 2x)dx = 0$

$$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$$

$$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow \text{Put } e^{-y} = z$$

$$\frac{-dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$$

$$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$$

$xe^{-y} = \frac{1}{2}\log_e x + c$, passes through $(e,1)$

$\Rightarrow C = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

7. Consider the two statements :

$(S1) : (p \to q) \vee (\sim q \to p)$ is a tautology.

$(S2) : (p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

Then :

(1) only (S1) is true.

(2) both (S1) and (S2) are false.

(3) both (S1) and (S2) are true.

(4) only (S2) is true.

Official Ans. by NTA (3)

Sol. $S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 = q \vee (\sim p \vee p) = qvt = t =$ tautology

$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$

= fallacy

8. The domain of the function $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

(1) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$ (2) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

(3) $\left(-\frac{1}{2}, \infty\right) - \{0\}$ (4) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

Official Ans. by NTA (4)

Sol. $\frac{1+x}{x} \in (-\infty, -1] \cup [1, \infty)$

$\frac{1}{x} \in (-\infty, -2] \cup [0, \infty)$

$x \in \left[-\frac{1}{2}, 0\right) \cup (0, \infty)$

$x \in \left[-\frac{1}{2}, \infty\right) - \{0\}$

9. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P\left(X \geq 5 \,\middle|\, X > 2\right)$ is :

(1) $\frac{125}{216}$ (2) $\frac{11}{36}$ (3) $\frac{5}{6}$ (4) $\frac{25}{36}$

Official Ans. by NTA (4)

Sol. $P(x \geq 5 \mid x > 2) = \frac{P(x \geq 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \ldots\ldots + \infty}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \ldots\ldots + \infty}$$

$$\frac{\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1 - \frac{5}{6}}}{\frac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1 - \frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

10. If $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$, then the value of tan p is :

(1) $\frac{101}{102}$ (2) $\frac{50}{51}$ (3) 100 (4) $\frac{51}{50}$

Official Ans. by NTA (2)

Sol. $\sum_{r=1}^{50} \tan^{-1}\left(\frac{2}{4r^2}\right) = \sum_{r=1}^{50} \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$\sum_{r=1}^{50} \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

$\tan^{-1}(101) - \tan^{-1}1 \Rightarrow \tan^{-1}\frac{50}{51}$

11. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$

$x + 2y + 3z = \mu$

$x + 3y + \lambda z = 1$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :

(1) $p = \frac{1}{6}$ and $q = \frac{1}{36}$ (2) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(3) $p = \frac{5}{6}$ and $q = \frac{1}{36}$ (4) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

Official Ans. by NTA (2)

Sol. $D \neq 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 5$

For no solution $D = 0 \Rightarrow \lambda = 5$

$$D_1 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} \neq 0 \Rightarrow \mu \neq 3$$

$$p = \frac{5}{6}$$

$$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

Option (2)

12. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :

(1) $y^3(x-2) = x^2$ (2) $x^3(x-2) = y^2$

(3) $y^2(x-2) = x^3$ (4) $x^2(x-2) = y^3$

Official Ans. by NTA (3)

Sol. $T = S_1$

$xh - yk = h^2 - k^2$

$$y = \frac{xh}{k} - \frac{\left(h^2 - k^2\right)}{k}$$

this touches $y^2 = 8x$ then $c = \frac{a}{m}$

$$\left(\frac{k^2 - h^2}{k}\right) = \frac{2k}{h}$$

$2y^2 = x(y^2 - x^2)$

$y^2(x-2) = x^3$

13. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

is :

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{4}$

(3) $\frac{1}{8}$ (4) $\frac{1}{8\sqrt{2}}$

Official Ans. by NTA (3)

Sol. $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$$

$$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{8}$$

14. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}\left(p+iq\right)$, then p and q are roots of the equation :

(1) $x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

(2) $x^2 + \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$

(3) $x^2 + \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

(4) $x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$

Official Ans. by NTA (1)

Sol. $\left(2e^{i\pi/6}\right)^{100} = 2^{99}\left(p+iq\right)$

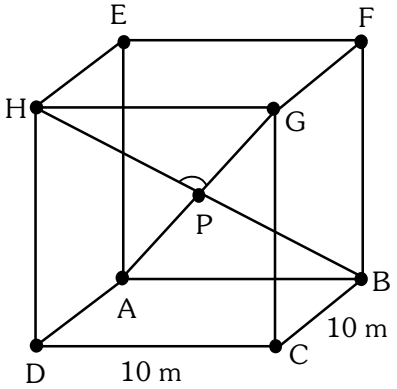
$$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}\left(p+iq\right)$$

$$p + iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$p = -1, q = \sqrt{3}$

$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0.$

15. A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :



(1) 5 (2) $2\sqrt{10}$

(3) $5\sqrt{3}$ (4) $5\sqrt{2}$

Official Ans. by NTA (4)

Sol. $A\left(\hat{j}\right)$. $B\left(10\hat{i}\right)$

H $\left(h\hat{j} + 10\hat{k}\right)$

G $\left(10\hat{i} + h\hat{j} + 10\hat{k}\right)$

$\overrightarrow{AG} = 10\hat{i} + h\hat{j} + 10\hat{k}$

$\overrightarrow{BH} = -10\hat{i} + h\hat{j} + 10\hat{k}$

$\cos\theta = \dfrac{\overrightarrow{AG}\,\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|}$

$\dfrac{1}{5} = \dfrac{h^2}{h^2 + 200}$

$4h^2 = 200 \Rightarrow h = 5\sqrt{2}$

16. Let P be the plane passing through the point (1,2,3) and the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+4\hat{k}\right)=16$ and $\vec{r}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)=6$. Then which of the following points does **NOT** lie on P ?

(1) (3, 3, 2) (2) (6, –6, 2)

(3) (4, 2, 2) (4) (–8, 8, 6)

Official Ans. by NTA (3)

Sol. $(x+y+4z-16)+\lambda(-x+y+z-6)=0$

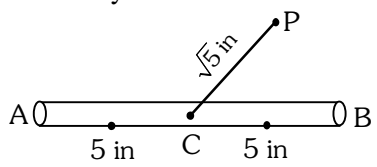
Passes through (1,2,3)

$-1+\lambda(-2) \Rightarrow \lambda = -\dfrac{1}{2}$

$2(x+y+4z-16)-(-x+y+z-6)=0$

$3x + y + 7z - 26 = 0$

17. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :

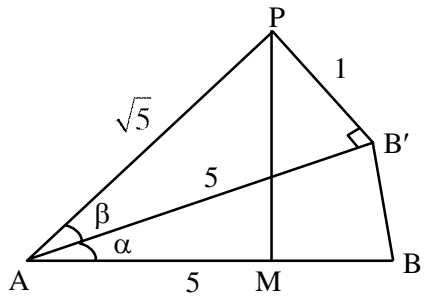


(1) $\tan^{-1}\left(\dfrac{3}{4}\right)$ (2) $\tan^{-1}(1)$

(3) $\tan^{-1}\left(\dfrac{4}{3}\right)$ (4) $\tan^{-1}\left(\dfrac{1}{2}\right)$

Official Ans. by NTA (1)

Sol.



From figure.

$\sin\beta = \dfrac{1}{\sqrt{5}}$

$\therefore \tan\beta = \dfrac{1}{2}$

$\tan(\alpha+\beta) = 2$

$\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta} = 2$

$\dfrac{\tan\alpha + \dfrac{1}{2}}{1 - \tan\alpha\left(\dfrac{1}{2}\right)} = 2$

$\tan\alpha = \dfrac{3}{4}$

$\boxed{\alpha = \tan^{1}\left(\dfrac{3}{4}\right)}$

18. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ is

(1) $\dfrac{\pi}{2}$ (2) $\dfrac{5\pi}{4}$

(3) $\dfrac{3\pi}{4}$ (4) $\dfrac{3\pi}{2}$

Official Ans. by NTA (3)

Sol. $I = \displaystyle\int_0^{\pi/2} \frac{(1+\sin^2 x)}{(1+\pi^{\sin x})} + \frac{\pi^{\sin x}(1+\sin^2 x)}{(1+\pi^{\sin x})}dx$

$I = \displaystyle\int_0^{\pi/2} (1+\sin^2 x)dx$

$I = \dfrac{\pi}{2} + \dfrac{\pi}{2}\cdot\dfrac{1}{2} = \dfrac{3\pi}{4}$

19. A circle C touches the line x = 2y at the point (2,1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

(1) $7\sqrt{5}$ (2) 15

(3) $\sqrt{285}$ (4) $4\sqrt{15}$

Official Ans. by NTA (1)

Sol. $(x-2)^2+(y-1)^2+\lambda(x-2y)=0$

$C: x^2+y^2+x(\lambda-4)+y(-2-2\lambda)+5=0$

$C_1: x^2+y^2+2y-5=0$

$S_1-S_2=0$ (Equation of PQ)

$(\lambda-4)x-(2\lambda+4)y+10=0$ Passes through $(0,-1)$

$\Rightarrow \lambda=-7$

$C: x^2+y^2-11x+12y+5=0$

$=\frac{\sqrt{245}}{4}$

Diometer $=7\sqrt{5}$

20. $\lim\limits_{x\to 2}\left(\sum\limits_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to :

(1) $\frac{9}{44}$ (2) $\frac{5}{24}$

(3) $\frac{1}{5}$ (4) $\frac{7}{36}$

Official Ans. by NTA (1)

Sol. $S=\lim\limits_{x\to 2}\sum\limits_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}$

$S=\sum\limits_{n=1}^{9}\frac{2}{4(n^2+3n+2)}=\frac{1}{2}\sum\limits_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$S=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{9}{44}$

SECTION-B

1. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _____.

Official Ans. by NTA (7744)

Sol1. 209, 220, 231, ..., 495

Sum $=\frac{27}{2}(209+495)=9504$

Number containing 1 at unit place $\underline{2}\ \underline{3}\ \underline{1}$, $\underline{3}\ \underline{4}\ \underline{1}$, $\underline{4}\ \underline{5}\ \underline{1}$

Number containing 1 at 10^{th} place $\underline{3}\ \underline{1}\ \underline{9}$, $\underline{4}\ \underline{1}\ \underline{8}$

Required $=9501-(231+341+451+319+418)$

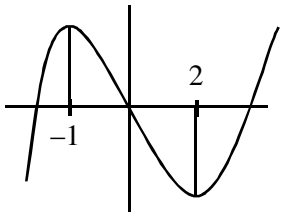
$=7744$

2. Let a and b respectively be the points of local maximum and local minimum of the function $f(x)=2x^3-3x^2-12x$. If A is the total area of the region bounded by $y=f(x)$, the x-axis and the lines $x=a$ and $x=b$, then 4A is equal to _____.

Official Ans. by NTA (114)

Sol. $f'(x)=6x^2-6x-12=6(x-2)(x+1)$

Point $=(2,-20)$ & $(-1,7)$



$A=\int\limits_{-1}^{0}(2x^3-3x^2-12x)dx+\int\limits_{0}^{2}(12x+3x^2-2x^3)dx$

$A=\left(\frac{x^4}{2}-x^3-6x^2\right)_{-1}^{0}+\left(6x^2+x^3-\frac{x^4}{2}\right)_{0}^{2}$

$4A=114$

3. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to _____.

Official Ans. by NTA (5)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=1,\ \vec{a}\cdot\vec{b}=12-\lambda$

$(\vec{a}\cdot\vec{b})=|\vec{b}|^2$

$\lambda^2-24\lambda+144=\lambda^2-4\lambda+4+40$

$20\lambda=100\Rightarrow\lambda=5.$

4. Let $a_1, a_2,....,a_{10}$ be an AP with common difference -3 and $b_1, b_2,...., b_{10}$ be a GP with common ratio 2. Let $c_k=a_k+b_k$, $k=1,2,...,10$. If $c_2=12$ and $c_3=13$, then $\sum\limits_{k=1}^{10}c_k$ is equal to _____.

Official Ans. by NTA (2021)

Sol. $c_2=a_2+b_2=a_1-3+2b_1=12$

$a_1 + 2b_1 = 15$ ____(1)

$c_3 = a_3 + b_3 = a_1 - 6 + 4b_1 = 13$

$a_1 + 4b_1 = 19$ ____(2)

from (1) & (2) $b_1 = 2$, $a_1 = 11$

$$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10}(a_k + b_k) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$$= \frac{10}{2}(2\times 11 + 9\times(-3)) + \frac{2(2^{10}-1)}{2-1}$$

$= 5(22 - 27) + 2(1023)$
$= 2046 - 25 = 2021$

5. Let Q be the foot of the perpendicular from the point P(7,–2,13) on the plane containing the lines $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$. Then $(PQ)^2$, is equal to _____.

Official Ans. by NTA (96)

Sol. Containing the line $\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 0$

$9(x + 1) - 18(y - 1) + 9(z - 3) = 0$

$x - 2y + z = 0$

$$PQ = \left|\frac{7+4+13}{\sqrt{6}}\right| = 4\sqrt{6}$$

$PQ^2 = 96$

6. Let $\binom{n}{k}$ denotes ${}^nC_k$ and $\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}$

If $A_k = \sum_{i=0}^{9}\binom{9}{i}\begin{bmatrix} 12 \\ 12-k+i \end{bmatrix} + \sum_{i=0}^{8}\binom{8}{i}\begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$

and $A_4 - A_3 = 190$ p, then p is equal to :

Official Ans. by NTA (49)

Sol. $A_k = \sum_{i=0}^{9} {}^9C_i \; {}^{12}C_{k-i} + \sum_{i=0}^{8} {}^8C_i \; {}^{13}C_{k-i}$

$A_k = {}^{21}C_k + {}^{21}C_k = 2\cdot {}^{21}C_k$

$A_4 - A_3 = 2({}^{21}C_4 - {}^{21}C_3) = 2(5985 - 1330)$

$190\,p = 2(5985 - 1330) \Rightarrow p = 49.$

7. Let $\lambda \neq 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _____.

Official Ans. by NTA (18)

Sol. $3\alpha^2 - 10\alpha + 27\lambda = 0$ _____(1)

$\alpha^2 - \alpha + 2\lambda = 0$ _____(2)

(1) – 3(2) gives

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (1) we get

$9\lambda^2 - 3\lambda + 2\lambda - 0$

$9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \neq 0$

Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$

$\alpha + \beta = 1 \Rightarrow \beta = 2/3$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}\times 3}{\frac{1}{9}} = 18$$

8. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x – y is _____.

Official Ans. by NTA (12)

Sol. $5 = \frac{3+7+x+y}{4} \Rightarrow x + y = 10$

$$\text{Var}(x) = 10 = \frac{3^2+7^2+x^2+y^2}{4} - 25$$

$140 = 49 + 9 + x^2 + y^2$

$x^2 + y^2 = 82$

$x + y = 10$

$\Rightarrow (x,y) = (9,1)$

Four numbers are 21,9,10,8

$$\text{Mean} = \frac{48}{4} = 12$$

9. Let A be a 3 × 3 real matrix.

If $\det(2\text{Adj}(2\ \text{Adj}(\text{Adj}(2A)))) = 2^{41}$, then the value of $\det(A^2)$ equal _____.

Official Ans. by NTA (4)

Sol. adj (2A) = 2^2 adjA

$\Rightarrow$ adj(adj (2A)) = adj(4 adjA) = 16 adj (adj A)

= 16 |A| A

$\Rightarrow$ adj (32 |A| A) = $(32\ |A|)^2$ adj A

$12(32|A|)^2\ |\text{adj } A| = 2^3\ (32|A|)^6\ |\text{adj } A|$

$2^3.2^{30}\ |A|^6\ .\ |A|^2 = 2^{41}$

$|A|^8 = 2^8 \Rightarrow |A| = \pm 2$

$|A|^2 = |A|^2 = 4$

10. The least positive integer n such that

$\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$ is a positive integer, is _____.

Official Ans. by NTA (6)

Sol. $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}}$

$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{2^{\frac{n+2}{2}}; i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$

This is positive integer for n = 6