EXERCISE 3.1

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

   (i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

   (ii) 5 pencils and 7 pens together cost 50, whereas 7 pencils and 5 pens together cost 46. Find the cost of one pencil and that of one pen.

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

   (i) $5x - 4y + 8 = 0$
   $7x + 6y - 9 = 0$

   (ii) $9x + 3y + 12 = 0$
   $18x + 6y + 24 = 0$

   (iii) $6x - 3y + 10 = 0$
   $2x - y + 9 = 0$

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

   (i) $3x + 2y = 5; 2x - 3y = 7$

   (ii) $2x - 3y = 8; 4x - 6y = 9$

   (iii) $\frac{3}{2}x + \frac{5}{3}y = 7; 9x - 10y = 14$

   (iv) $5x - 3y = 11; -10x + 6y = -22$

   (v) $\frac{4}{3}x + 2y = 8; 2x + 3y = 12$

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

   (i) $x + y = 5,$ $2x + 2y = 10$

   (ii) $x - y = 8,$ $3x - 3y = 16$

   (iii) $2x + y - 6 = 0,$ $4x - 2y - 4 = 0$

   (iv) $2x - 2y - 2 = 0,$ $4x - 4y - 5 = 0$

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.
6. Given the linear equation $2x+3y-8=0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
   (i) intersecting lines (ii) parallel lines
   (iii) coincident lines
7. Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$.
   Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Solutions:

Step 1: Form the equations. Let ' x ' be the number of girls and ' y ' be the number of boys.

- Equation 1 (Total students): The total number of students is 10. $x+y=10$
- Equation 2 (Relationship between girls and boys): The number of girls is 4 more than the number of boys. $x=y+4$ or $x-y=4$

Step 2: Find the solution graphically. To plot these lines, let's find a few points for each equation.

- For Equation 1: $x+y=10$
  - If $x=0$, then $y=10$. (Point: $(0,10)$)
  - If $x=10$, then $y=0$. (Point: $(10,0)$)
  - If $x=5$, then $y=5$. (Point: $(5,5)$)
- For Equation 2: $x-y=4$
  - If $x=4$, then $y=0$. (Point: $(4,0)$)
  - If $x=0$, then $y=-4$. (Point: $(0,-4)$)
  - If $x=7$, then $y=3$. (Point: $(7,3)$)

Plot these points and draw the lines. The point where the two lines intersect is the solution.

The lines intersect at the point $(7,3)$.

Answer:

- The number of girls (x) is 7.
- The number of boys (y) is 3.

You can check this: 7+3=10 (correct) and 7=3+4 (correct).

Problem 1 (ii): The Pencils and Pens

Step 1: Form the equations. Let ' x ' be the cost of one pencil and ' y ' be the cost of one pen.

- Equation 1: 5 pencils and 7 pens cost ₹ $50.5x+7y=50$
- Equation 2: 7 pencils and 5 pens cost ₹ $46.7x+5y=46$

Step 2: Find the solution graphically. We'll find points for each equation to plot the lines.

- **For Equation 1**: $5\mathbf{x}+7\mathbf{y}=50$
  - If we take $x=3,5(3)+7y=50 \Rightarrow 15+7y=50 \Rightarrow 7y=35 \Rightarrow y=5$. (Point: $(3,5)$)
  - If we take $x=10,5(10)+7y=50 \Rightarrow 50+7y=50 \Rightarrow 7y=0 \Rightarrow y=0$. (Point: $(10,0)$)
- **For Equation 2:** $7x+5y=46$
  - If we take $x=3,7(3)+5y=46 \Rightarrow 21+5y=46 \Rightarrow 5y=25 \Rightarrow y=5$. (Point: $(3,5)$)
  - If we take $x=8,7(8)+5y=46 \Rightarrow 56+5y=46 \Rightarrow 5y=-10 \Rightarrow y=-2$. (Point: $(8,-2)$)

Plot these points and draw the lines. The intersection point is the solution.

The lines intersect at the point $(3,5)$.

Answer:

- The cost of one pencil (x) is ₹3.
- The cost of one pen (y) is ₹5.

You can check this: $5(3)+7(5)=15+35=50$ (correct) and

$7(3)+5(5)=21+25=46$ (correct).

Question 2: Nature of Lines

For a pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, the nature of the lines can be determined by comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$, and $\frac{c_1}{c_2}$.

- If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the lines intersect at a unique point.
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the lines are coincident (they are the same line).
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel.

(i) $5x-4y+8=0$ and $7x+6y-9=0$

$$\frac{a_1}{a_2}=\frac{5}{7}$$

$$\frac{b_1}{b_2}=\frac{-4}{6}=\frac{-2}{3}$$

Since $\frac{5}{7} \neq \frac{-2}{3}$, the lines intersect at a point.

(ii) $9x+3y+12=0$ and $18x+6y+24=0$

$$\frac{a_1}{a_2}=\frac{9}{18}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}$$

$$\frac{c_1}{c_2}=\frac{12}{24}=\frac{1}{2}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, the lines are coincident.

(iii) $6x-3y+10=0$ and $2x-y+9=0$

$$\frac{a_1}{a_2} = \frac{6}{2} = 3$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = 3$$

$$\frac{c_1}{c_2} = \frac{10}{9}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_3} \neq \frac{c_1}{c_9}$, the lines are parallel.

Question 3: Consistent or Inconsistent

A pair of linear equations is consistent if it has at least one solution (intersecting or coincident lines). It is inconsistent if it has no solution (parallel lines).

Consistent: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Inconsistent: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(i) $3x + 2y = 5$ **and** $2x - 3y = 7$

$\frac{a_1}{a_2} = \frac{3}{2}$ and $\frac{b_1}{b_2} = \frac{2}{-3}$

Since $\frac{3}{2} \neq \frac{2}{-3}$, the pair of equations is consistent.

(ii) $2x - 3y = 8$ and $4x - 6y = 9$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{8}{9}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the pair of equations is inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$ and $9x - 10y = 14$

$$\frac{a_1}{a_2} = \frac{3/2}{9} = \frac{3}{18} = \frac{1}{6}$$

$$\frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{5}{-30} = -\frac{1}{6}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations is consistent.

(iv) $5x - 3y = 11$ and $-10x + 6y = -22$

$$\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{11}{-22} = -\frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equations is consistent.

(v) $\frac{4}{3}x + 2y = 8$ and $2x + 3y = 12$

$$\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{b_1}{b_2} = \frac{2}{3}$$

$$\frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equations is consistent.

Question 4: Consistent and Graphical Solution

First, determine consistency using the ratios. If the equations are consistent, find the graphical solution.

First, determine consistency using the ratios. If the equations are consistent, find the graphical solution.

(i) $x + y = 5$ and $2x + 2y = 10$

$\frac{a_1}{a_2}=\frac{1}{2},\frac{b_1}{b_2}=\frac{1}{2},\frac{c_1}{c_2}=\frac{5}{10}=\frac{1}{2}$.

Since the ratios are equal, the lines are **coincident**, which means the equations are consistent and have infinitely many solutions. Both equations represent the same line.

(ii) $x-y=8$ and $3x-3y=16$

$\frac{a_1}{a_2}=\frac{1}{3},\frac{b_1}{b_2}=\frac{-1}{-3}=\frac{1}{3},\frac{c_1}{c_2}=\frac{8}{16}=\frac{1}{2}$.

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, the lines are parallel. The equations are inconsistent and have no solution.

(iii) $2x+y-6=0$ and $4x-2y-4=0$

$$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{1}{-2}=-\frac{1}{2}$$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$, the lines **intersect** at a point. The equations are **consistent.**

To find the solution graphically, we plot the two lines.

For $2x+y=6$: Points are $(0,6),(3,0),(2,2)$.

For $4x-2y=4$: Points are $(1,0),(0,-2),(2,2)$.

The lines intersect at $(2,2)$.

(iv) $2x-2y-2=0$ and $4x-4y-5=0$

$$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}$$

$$\frac{b_1}{b_2}=\frac{-2}{-4}=\frac{1}{2}$$

$$\frac{c_1}{c_2}=\frac{-2}{-5}=\frac{2}{5}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel. The equations are inconsistent and have no solution.

Question 5: Dimensions of a Garden

Let the length of the garden be ' l ' and the width be ' w '.

- **Condition 1:** The length is 4 m more than the width.
  $l = w + 4$
- **Condition 2:** Half the perimeter is 36 m. The perimeter of a rectangle is $2(l + w)$.
  $\frac{1}{2} \times 2(l + w) = 36 \Rightarrow l + w = 36$
  We have two equations:
  $l - w = 4$
  $l + w = 36$
  Adding the two equations: $2l = 40 \Rightarrow l = 20$.
  Substituting $l = 20$ into the second equation: $20 + w = 36 \Rightarrow w = 16$.
  The dimensions of the garden are: length = 20 m and width = 16 m.

Question 6: Writing a Second Linear Equation

Given the equation $2x + 3y - 8 = 0$, we need to find another linear equation that satisfies a given condition.

(i) **Intersecting lines:** The condition is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

We can choose an equation where the ratio of the coefficients is different.

Example: $3x + 2y - 10 = 0$. Here, $\frac{2}{3} \neq \frac{3}{2}$.

(ii) Parallel lines: The condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

We can multiply the coefficients of x and y by a constant, but change the constant term.

Example: $4x+6y-15=0$. Here, $\frac{2}{4}=\frac{3}{6}\neq\frac{-8}{-15}$.

(iii) Coincident lines: The condition is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

We can multiply the entire equation by a constant.

Example: $4x+6y-16=0$. This is just $2(2x+3y-8)=0$.

Question 7: Triangle Formed by Lines and the X -axis

We need to graph the two lines and find the vertices of the triangle they form with the x-axis.

Line 1: $x-y+1=0 \Rightarrow y=x+1$

Line 2: $3x+2y-12=0 \Rightarrow 2y=12-3x \Rightarrow y=6-\frac{3}{2}x$

To find the vertices, we need the intersection points:

1. **Intersection of Line 1 and Line 2 :**

   $x+1=6-\frac{3}{2}x \Rightarrow x+\frac{3}{2}x=5 \Rightarrow \frac{5}{2}x=5 \Rightarrow x=2$.

   Substitute $x=2$ into $y=x+1$, we get $y=3$.

   Vertex 1: $(2,3)$

2. **Intersection of Line 1 with the $\mathbf{x}$-axis (where $\mathbf{y}=0$):**

   $0=x+1 \Rightarrow x=-1$.

   Vertex 2: (-1,0)

3. **Intersection of Line 2 with the $\mathbf{x}$-axis (where $\mathbf{y}=0$):**

   $0=6-\frac{3}{2}x \Rightarrow \frac{3}{2}x=6 \Rightarrow x=4$.

   Vertex 3: $(4,0)$

   The vertices of the triangle are $(2,3),(-1,0)$, and $(4,0)$.

EXERCISE 3.2

1. Solve the following pair of linear equations by the substitution method.

   (i) $x+y=14$ (ii) $s-t=3$

$x - y = 4$ $\frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3$ (iv) $0.2x + 0.3y = 1.3$

$9x - 3y = 9$ $0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$ (vi) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\sqrt{3}x - \sqrt{8}y = 0$ $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of ' m ' for which $y = mx + 3$.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km , the charge paid is ₹ 105 and for a journey of 15 km , the charge paid is ₹ 155. What are the fixed charges and the charge per km ? How much does a person have to pay for travelling a distance of 25 km ?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Question 1: Solving by Substitution

The substitution method involves these steps:

1. **Express one variable** in terms of the other from one of the equations.
2. **Substitute this expression** into the second equation to get a linear equation in a single variable.
3. **Solve** for that variable.
4. **Substitute the value back** into the expression from Step 1 to find the other variable.

(i) $x+y=14$ and $x-y=4$

From the first equation, $x=14-y$.

Substitute into the second equation:

$(14-y)-y=4 \Rightarrow 14-2y=4 \Rightarrow -2y=-10 \Rightarrow y=5$.

Now find $x: x=14-5=9$.

Solution: $x=9, y=5$.

(ii) $s-t=3$ and $\frac{s}{3}+\frac{t}{2}=6$

From the first equation, $s=3+t$.

Substitute into the second equation: $\frac{3+t}{3}+\frac{t}{2}=6$.

To eliminate the denominators, multiply the whole equation by 6:

$2(3+t)+3t=36 \Rightarrow 6+2t+3t=36 \Rightarrow 5t=30 \Rightarrow t=6$.

Now find $s: s=3+6=9$.

Solution: $s=9, t=6$.

(iii) $3x-y=3$ and $9x-3y=9$

From the first equation, $y=3x-3$.

Substitute into the second equation: $9x-3(3x-3)=9 \Rightarrow 9x-9x+9=9 \Rightarrow 9=9$.

This is a true statement. It means the equations are coincident and have infinitely many

Solutions:

(iv) $0.2x+0.3y=1.3$ and $0.4x+0.5y=2.3$

Multiply both equations by 10 to remove decimals:

$2x+3y=13$

$4x+5y=23$

From the first equation, $2x=13-3y \Rightarrow x=\frac{13-3y}{2}$.

Substitute into the second equation:

$4\left(\frac{13-3y}{2}\right)+5y=23 \Rightarrow 2(13-3y)+5y=23 \Rightarrow 26-6y+5y=23 \Rightarrow -y=-3 \Rightarrow y=3$.

Now find $x: x=\frac{13-3(3)}{2}=\frac{13-9}{2}=\frac{4}{2}=2$.

Solution: $x=2, y=3$.

(v) $\sqrt{2}x+\sqrt{3}y=0$ and $\sqrt{3}x-\sqrt{8}y=0$

From the first equation, $\sqrt{2}x=-\sqrt{3}y \Rightarrow x=-\frac{\sqrt{3}}{\sqrt{2}}y$.

Substitute into the second equation: $\sqrt{3}\left(-\frac{\sqrt{3}}{\sqrt{2}}y\right)-\sqrt{8}y=0 \Rightarrow -\frac{3}{\sqrt{2}}y-2\sqrt{2}y=0$.

Multiply by $\sqrt{2}: -3y-2(\sqrt{2})^2y=0 \Rightarrow -3y-4y=0 \Rightarrow -7y=0 \Rightarrow y=0$.

Now find $x: x=-\frac{\sqrt{3}}{\sqrt{2}}(0)=0$.

Solution: $x=0, y=0$.

(v) $\sqrt{2}x+\sqrt{3}y=0$ and $\sqrt{3}x-\sqrt{8}y=0$

From the first equation, $\sqrt{2}x=-\sqrt{3}y \Rightarrow x=-\frac{\sqrt{3}}{\sqrt{2}}y$.

Substitute into the second equation: $\sqrt{3}\left(-\frac{\sqrt{3}}{\sqrt{2}}y\right)-\sqrt{8}y=0 \Rightarrow -\frac{3}{\sqrt{2}}y-2\sqrt{2}y=0$.

Multiply by $\sqrt{2}$: $-3y-2(\sqrt{2})^2y=0 \Rightarrow -3y-4y=0 \Rightarrow -7y=0 \Rightarrow y=0$.

Now find $x: x=-\frac{\sqrt{3}}{\sqrt{2}}(0)=0$.

Solution: $x=0, y=0$.

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$ and $\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

First, simplify the equations by clearing the fractions:

Multiply first equation by 6: $9x-10y=-12$.

Multiply second equation by 6: $2x+3y=13$.

From the second equation, $2x=13-3y \Rightarrow x=\frac{13-3y}{2}$.

Substitute into the first equation: $9\left(\frac{13-3y}{2}\right)-10y=-12$.

Multiply by 2:

$9(13-3y)-20y=-24 \Rightarrow 117-27y-20y=-24 \Rightarrow -47y=-141 \Rightarrow y=3$.

Now find $x: x=\frac{13-3(3)}{2}=\frac{13-9}{2}=\frac{4}{2}=2$.

Solution: $x=2, y=3$.

Question 2: Solving and Finding ' m '

Given the equations $2x+3y=11$ and $2x-4y=-24$.

From the first equation, $2x=11-3y$.

Substitute into the second equation:

$(11-3y)-4y=-24 \Rightarrow 11-7y=-24 \Rightarrow -7y=-35 \Rightarrow y=5$.

Now find $x: 2x=11-3(5) \Rightarrow 2x=11-15 \Rightarrow 2x=-4 \Rightarrow x=-2$.

The solution is $x=-2$ and $y=5$.

We need to find ' m ' for the equation $y=mx+3$.

$5=m(-2)+3 \Rightarrow 5=-2m+3 \Rightarrow 2=-2m \Rightarrow m=-1$.

Answer: The value of m is -1.

Question 3: Word Problems

(i) Difference between two numbers.

Let the two numbers be x and y.

- $x-y=26$

- $x = 3y$

Substitute the second equation into the first: $3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13$.

Now find $x : x = 3(13) = 39$.

Numbers: 39 and 13.

(ii) Supplementary angles.

Let the two supplementary angles be x and y, where $x > y$.

- Supplementary angles add up to $180^\circ : x + y = 180$
- The larger exceeds the smaller by 18 degrees: $x = y + 18$

Substitute the second equation into the first: $(y + 18) + y = 180 \Rightarrow 2y = 162 \Rightarrow y =$ 81.

Now find $x : x = 81 + 18 = 99$.

Angles: 99° and 81°.

(iii) Cricket bats and balls.

Let the cost of one bat be x and one ball be y.

- $7x + 6y = 3800$
- $3x + 5y = 1750$

From the second equation, $3x = 1750 - 5y \Rightarrow x = \frac{1750 - 5y}{3}$.

Substitute into the first equation: $7\left(\frac{1750 - 5y}{3}\right) + 6y = 3800$.

Multiply by 3: $7(1750 - 5y) + 18y = 11400 \Rightarrow 12250 - 35y + 18y = 11400$.

$\Rightarrow -17y = -850 \Rightarrow y = 50$

Now find $x : x = \frac{1750 - 5(50)}{3} = \frac{1750 - 250}{3} = \frac{1500}{3} = 500$.

Cost: One bat costs ₹500 and one ball costs ₹50.

(iv) **Taxi charges**.

Let the fixed charge be x and the charge per km be y.

- Journey of $10 \text{ km} : x + 10y = 105$
- Journey of $15 \text{ km} : x + 15y = 155$

From the first equation, $x = 105 - 10y$.

Substitute into the second equation:

$(105 - 10y) + 15y = 155 \Rightarrow 105 + 5y = 155 \Rightarrow 5y = 50 \Rightarrow y = 10$.

Now find $x : x = 105 - 10(10) = 5$.

- **Fixed charge:** ₹5.
- **Charge per km:** ₹10.
- **Charge for 25 km:** $x + 25y = 5 + 25(10) = 5 + 250 = 255$.

Answer: A person has to pay ₹255 for travelling 25 km .

(v) Fraction.

Let the fraction be $\frac{x}{y}$.

- If 2 is added to both numerator and denominator, the fraction becomes $\frac{9}{11}$

  . $\frac{x+2}{y+2} = \frac{9}{11} \Rightarrow 11(x+2) = 9(y+2) \Rightarrow 11x + 22 = 9y + 18 \Rightarrow 11x - 9y = -4$

- If 3 is added to both, it becomes $\frac{5}{6}$.

  $\frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 6(x+3) = 5(y+3) \Rightarrow 6x + 18 = 5y + 15 \Rightarrow 6x - 5y = -3$

  From the second equation, $6x = 5y - 3 \Rightarrow x = \frac{5y-3}{6}$.

  Substitute into the first equation: $11\left(\frac{5y-3}{6}\right) - 9y = -4$.

  Multiply by 6: $11(5y-3) - 54y = -24 \Rightarrow 55y - 33 - 54y = -24$.

  $\Rightarrow y - 33 = -24 \Rightarrow y = 9$

  Now find $x : x = \frac{5(9)-3}{6} = \frac{45-3}{6} = \frac{42}{6} = 7$.

  Answer: The fraction is $\frac{7}{9}$.

(vi) Ages of Jacob and his son.

Let Jacob's present age be x and his son's be y.

- **5 years hence (in 5 years):**

  Jacob's age will be $x+5$, son's age will be $y+5$.

  $x+5=3(y+5) \Rightarrow x+5=3y+15 \Rightarrow x-3y=10$

- **5 years ago:**

  Jacob's age was $x-5$, son's age was $y-5$.

  $x-5=7(y-5) \Rightarrow x-5=7y-35 \Rightarrow x-7y=-30$

  From the first equation, $x=3y+10$.

  Substitute into the second equation: $(3y+10)-7y=-30 \Rightarrow -4y=-40$.

  $\Rightarrow y=10$

  Now find $x: x=3(10)+10=40$.

  Answer: Jacob's present age is 40 years and his son's is 10 years.

EXERCISE 3.3

1. Solve the following pair of linear equations by the elimination method and the substitution method:

   (i) $x+y=5$ and $2x-3y=4$ (ii) $3x+4y=10$ and $2x-2y=2$

   (iii) $3x-5y-4=0$ and $9x=2y+7$ (iv) $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Question 1: Solving Linear Equations

(i) $x+y=5$ (Equation 1)

$2x-3y=4$ (Equation 2)

- **Elimination Method:**

Multiply Equation 1 by 2 to make the coefficient of x the same as in Equation 2:

$2(x+y)=2(5)\Rightarrow 2x+2y=10$ (Equation 3).

Subtract Equation 2 from Equation 3:

$(2x+2y)-(2x-3y)=10-4$

$5y=6\Rightarrow y=\frac{6}{5}$

Substitute the value of y back into Equation 1:

$x+\frac{6}{5}=5\Rightarrow x=5-\frac{6}{5}\Rightarrow x=\frac{25-6}{5}=\frac{19}{5}$.

Solution: $x=\frac{19}{5}, y=\frac{6}{5}$.

- **Substitution Method:**

From Equation 1, express x in terms of $y: x=5-y$.

Substitute this into Equation 2:

$2(5-y)-3y=4\Rightarrow 10-2y-3y=4\Rightarrow 10-5y=4\Rightarrow -5y=-6\Rightarrow y=\frac{6}{5}$.

Substitute $y=\frac{6}{5}$ back to find $x: x=5-\frac{6}{5}=\frac{19}{5}$.

Solution: $x=\frac{19}{5}, y=\frac{6}{5}$.

(ii) $3x+4y=10$ (Equation 1)

$2x-2y=2$ (Equation 2)

• **Elimination Method:**

Multiply Equation 2 by 2 to eliminate $y: 2(2x-2y)=2(2) \Rightarrow 4x-4y=4$ (Equation 3).

Add Equation 1 and Equation 3:

$$(3x+4y)+(4x-4y)=10+4$$

$$7x=14 \Rightarrow x=2$$

Substitute $x=2$ back into Equation 1:

$3(2)+4y=10 \Rightarrow 6+4y=10 \Rightarrow 4y=4 \Rightarrow y=1$.

Solution: $x=2, y=1$.

- Substitution Method:

From Equation 2, express x in terms of $y: 2x=2+2y \Rightarrow x=1+y$.

Substitute this into Equation 1:

$3(1+y)+4y=10 \Rightarrow 3+3y+4y=10 \Rightarrow 7y=7 \Rightarrow y=1$.

Substitute $y=1$ back to find x : $x=1+1=2$.

Solution: $x=2, y=1$.

(iii) $3x-5y-4=0 \Rightarrow 3x-5y=4$ (Equation 1)

$9x=2y+7 \Rightarrow 9x-2y=7$ (Equation 2)

•**Elimination Method:**

Multiply Equation 1 by 3 to eliminate $x: 3(3x-5y)=3(4) \Rightarrow 9x-15y=12$ (Equation $3)$.

Subtract Equation 2 from Equation 3:

$$(9x-15y)-(9x-2y)=12-7$$

$$-13y=5 \Rightarrow y=-\frac{5}{13}$$

Substitute $y = -\frac{5}{13}$ back into Equation 1:

$$3x - 5\left(-\frac{5}{13}\right) = 4 \Rightarrow 3x + \frac{25}{13} = 4 \Rightarrow 3x = 4 - \frac{25}{13} \Rightarrow 3x = \frac{52-25}{13} = \frac{27}{13} \Rightarrow x = \frac{9}{13}.$$

Solution: $x = \frac{9}{13}, y = -\frac{5}{13}$.

(iv) $\frac{x}{2} + \frac{2y}{3} = -1 \Rightarrow 3x + 4y = -6$ (Equation 1, after multiplying by 6)

$x - \frac{y}{3} = 3 \Rightarrow 3x - y = 9$ (Equation 2, after multiplying by 3)

• Elimination Method:

Subtract Equation 2 from Equation 1:

$$(3x + 4y) - (3x - y) = -6 - 9$$

$$5y = -15 \Rightarrow y = -3$$

Substitute $y = -3$ back into Equation 2:

$$3x - (-3) = 9 \Rightarrow 3x + 3 = 9 \Rightarrow 3x = 6 \Rightarrow x = 2.$$

Solution: $x = 2, y = -3$.

Question 2: Word Problems (Elimination Method)

(i) Fraction.

Let the fraction be $\frac{x}{y}$.

- **Condition 1:** Add 1 to numerator, subtract 1 from denominator, fraction becomes 1.

  $$\frac{x+1}{y-1} = 1 \Rightarrow x + 1 = y - 1 \Rightarrow x - y = -2 \text{ (Equation 1)}$$

- **Condition 2:** If we only add 1 to the denominator, fraction becomes $\frac{1}{2}$.

  $$\frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x = y + 1 \Rightarrow 2x - y = 1 \text{ (Equation 2)}$$

Subtract Equation 1 from Equation 2:

$$(2x-y)-(x-y)=1-(-2)$$

$$x=3$$

Substitute $x=3$ back into Equation 1: $3-y=-2 \Rightarrow -y=-5 \Rightarrow y=5$.

Answer: The fraction is $\frac{3}{5}$.

(ii) Ages of Nuri and Sonu.

Let Nuri's age be x and Sonu's age be y.

- **5 years ago:**

Nuri's age: $x-5$. Sonu's age: $y-5$.

$x-5=3(y-5) \Rightarrow x-5=3y-15 \Rightarrow x-3y=-10$ (Equation 1)

- 10 years later:

Nuri's age: $x+10$. Sonu's age: $y+10$.

$x+10=2(y+10) \Rightarrow x+10=2y+20 \Rightarrow x-2y=10$ (Equation 2)

Subtract Equation 1 from Equation 2:

$$(x-2y)-(x-3y)=10-(-10)$$

$$y=20$$

Substitute $y=20$ into Equation 2: $x-2(20)=10 \Rightarrow x-40=10 \Rightarrow x=50$.

Answer: Nuri is 50 **years old** and Sonu is 20 **years old.**

(iii) Two-digit number.

Let the two-digit number be $10x+y$, where x is the tens digit and y is the units digit. The sum of the digits is 9 , so $x+y=9$.

- **Condition 1:** The sum of the digits is 9 .

  $x+y=9$ (Equation 1)

- **Condition 2:** 9 times the number is twice the number obtained by reversing the digits.

The reversed number is $10y+x$.

$$9(10x+y)=2(10y+x)$$

$$90x+9y=20y+2x$$

$$88x-11y=0 \Rightarrow 8x-y=0 \text{ (Equation 2)}$$

Add Equation 1 and Equation 2:

$$(x+y)+(8x-y)=9+0$$

$$9x=9 \Rightarrow x=1$$

Substitute $x=1$ into Equation 1: $1+y=9 \Rightarrow y=8$.

The digits are $x=1$ and $y=8$.

Answer: The number is 18.

(iv) Meena's bank withdrawal.

Let the number of ₹50 notes be x and the number of ₹100 notes be y.

- **Condition 1:** Total notes are 25.

  $x+y=25$ (Equation 1)

- Condition 2: Total value is ₹2000.

  $50x+100y=2000 \Rightarrow x+2y=40$ (Equation 2, after dividing by 50)

Subtract Equation 1 from Equation 2:

$$(x+2y)-(x+y)=40-25$$

$$y=15$$

Substitute $y=15$ into Equation 1: $x+15=25 \Rightarrow x=10$.

Answer: Meena received 10 notes of ₹50 and 15 notes of ₹100.

(v) Library charges.

Let the fixed charge for the first three days be x and the additional charge per day be y.

- **Condition 1:** Saritha paid ₹27 for a book kept for 7 days.

  (7 days $=3$ fixed days +4 additional days)

  $x+4y=27$ (Equation 1)

- **Condition 2:** Susy paid ₹21 for a book kept for 5 days.

  (5 days $=3$ fixed days +2 additional days)

  $x+2y=21$ (Equation 2)

Subtract Equation 2 from Equation 1:

$$(x+4y)-(x+2y)=27-21$$

$$2y=6 \Rightarrow y=3$$

Substitute $y=3$ into Equation 2: $x+2(3)=21 \Rightarrow x+6=21 \Rightarrow x=15$.

Answer: The fixed charge is **₹15** and the additional charge per day is ₹3.